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Shadow area growth with increasing of charge and angular momentum in the Kerr-Newman metric with irreducible mass

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Abstract

The standard form of the Kerr-Newman (KN) metric contains the total mass of a charged rotating source, which includes the mass of the neutral non-rotating source (the "irreducible" mass), as well as the mass equivalents of the rotational and electric field energies. According to the Christodolou-Ruffini mass formula, at a constant irreducible mass, the total mass increases with increasing angular momentum and charge. However, ignoring this fact, at studying the dependence of the effects of gravity on parameters, the total mass was assumed to be constant with varying angular momentum and/or charge, and this error led to physically absurd results, as if an increase in charge and/or angular momentum, by increasing the energy of the source, weakens the effects of gravity. Recently, the author introduced the concept of metrics with independent parameters and formulated a new method for describing the effects of gravity based on it, leading to physically correct results (Z. Zakir 2022). The KN metric was also expressed through independent parameters, in particular through the irreducible mass, independent of charge and angular momentum (Z. Zakir 2023). It was shown that in the KN metric with irreducible mass, an increase in charge and angular momentum does not weaken the effects of gravity, as was previously accepted, but, on the contrary, enhances all these effects, as it should be when the source energy increases. This letter presents results for the shadow in the KN metric with irreducible mass. In the figures 9 shadow contours with three values of each of parameters are presented. It is shown that, as for other gravity effects, an increase in charge and/or angular momentum, increasing the source's gravity, enhances this effect also, which is expressed in an increase in the shadow area.

Keywords: charged star, rotating star, Christodolou-Ruffini mass formula, total mass, metrics with independent parameters

The standard Kerr-Newman (KN) metric outside a charged rotating body contains its total mass M, determined from the transition to the Schwarzschild metric at $r\to\infty$. In the equatorial plane $\theta=\pi/2$ (in Boyer-Lindquist coordinates) it has the form (notations and references see in [1,2]):

$$ds^2=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2-\frac{r^2}{\Delta}dr^2+\frac{2a}{r}\left(2M-\frac{Q^2}{r}\right)d\varphi dt-$$
$$-\left[r^2+a^2+\frac{a^2}{r}\left(2M-\frac{Q^2}{r}\right)\right]d\varphi^2,\quad \Delta=r^2+a^2+Q^2-2Mr. \tag{1}$$

Here the total mass M contains the mass M_0 of the source's matter in a state without charge and angular momentum, as well as mass equivalents of the energies of the electric field and rotation. Therefore, M depends on three independent parameters - the mass of a neutral

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non-rotating body M_0, charge Q and rotation parameter $a = J / M$ (angular momentum per unit mass of the source). This means that the total mass included in the KN metric M is not an independent parameter and, at constant M_0, it changes when each of the other two parameters a and Q changes.

Various definitions of the dependence of M on a and Q are possible, differing in model assumptions about the source and its environment [1-6]. The well-known one, which has become generally accepted, is the Christodolou-Ruffini (CR) mass formula [3,4]:

$$M^2 = \left(M_{ir} + \frac{Q^2}{4M_{ir}} \right)^2 + \frac{J^2}{4M_{ir}^2}, \tag{2}$$

where M_{ir} is the "irreducible mass", defined in [3] as the residual mass of the source after removing its charge and angular momentum in a specific way. In the first approximation, M_{ir} is equal to M_0 and $M_{ir} \simeq M_0$. Substituting the expression for the angular momentum of the source $J = Ma$ into the CR formula (2) allows one to write (2) in a form $M = f(M_{ir}, a, Q)$ more convenient for applications, when M is expressed only through mutually independent parameters [6]:

$$M = M_{ir} \frac{1 + Q^2 / 4M_{ir}^2}{\sqrt{1 - a^2 / 4M_{ir}^2}}, \qquad J = Ma. \tag{3}$$

Notice that there is a weak indirect dependence of a on M_{ir} and Q, but in this letter we will neglect such a fine structure of the "spectrum" of the source, assuming that M_{ir}, Q and a are independent parameters within the accuracy sufficient for astrophysics.

Although the CR formula (2) was obtained using hypothetical methods for extracting the rotational energy of a body, nevertheless, qualitatively the course of the dependence M on the other two parameters is conveyed correctly by it, and the small differences from other mass formulas in taking into account the rotational energy are purely quantitative and quite small [1,2,6].

Therefore, in [6] to express the KN metric in terms of independent parameters in the general case, the CR mass formula (2) was used and a KN metric with irreducible mass was obtained. In particular, by substituting the mass formula (3) into (1), the KN metric with irreducible mass on the equatorial plane was obtained:

$$\begin{aligned} ds^2 = & \left(1 - \frac{2M_{ir}}{r} \frac{1 + Q^2 / 4M_{ir}^2}{\sqrt{1 - a^2 / 4M_{ir}^2}} + \frac{Q^2}{r^2} \right) dt^2 - \frac{r^2}{\Delta_{ir}} dr^2 - (r^2 + a^2) d\varphi^2 - \\ & - \frac{a^2}{r} \left(2M_{ir} \frac{1 + Q^2 / 4M_{ir}^2}{\sqrt{1 - a^2 / 4M_{ir}^2}} - \frac{Q^2}{r} \right) d\varphi^2 + \frac{2a}{r} \left(2M_{ir} \frac{1 + Q^2 / 4M_{ir}^2}{\sqrt{1 - a^2 / 4M_{ir}^2}} - \frac{Q^2}{r} \right) d\varphi dt, \end{aligned} \tag{4}$$

$$\Delta_{ir} = r^2 + a^2 + Q^2 - 2rM_{ir} \frac{1 + Q^2 / 4M_{ir}^2}{\sqrt{1 - a^2 / 4M_{ir}^2}}. \tag{5}$$

In the present letter, where the parametric dependence of the contour of the shadow of a charged rotating source is studied, this form of the KN metric will be used. A more detailed description and discussion will be given in [1,2]. Here only the final formula for the shadow contour and the resulting figures in the standard (1) and new (4) forms of the KN metric will be presented.

At constructing the contour of the shadow of a compact source of the KN metric, we will follow the article by A. de Vries in 2000 [7] (for a review of the literature, see [8,9]), where one of the first detailed studies of the shadow in the standard form of the KN metric (1) was given. The contour of the shadow on the equatorial plane is drawn on a plane perpendicular to the axis connecting the observer and the center of the source. By denoting the abscissa on this plane as α, and the ordinate as β, and solving the equations of motion for photons in the KN metric, the following expressions were obtained for the coordinates of the shadow contour points [7]:

$$\alpha(r) = \frac{M(r^2 - a^2) - rQ^2 - r(r^2 + a^2 + Q^2 - 2Mr)}{a(M - r)}, \tag{6}$$

$$\beta(r) = \pm \frac{r\sqrt{4a^2(Mr - Q^2) - [r(r - 3M) + 2Q^2]^2}}{a(M - r)}. \tag{7}$$

Considering this system of equations as a parametric form of a function $\beta(\alpha)$, where the parameter is r, and drawing its plot, one can obtain the contour of the shadow.

In [7], shadow contours were constructed using the standard form of the KN metric (1) and at changing a and Q it was supposed that the total mass is constant $M = const$. Plots of 16 contours were shown for four values of each of two parameters: $a, Q = 0,\ 1/2,\ \sqrt{3}/2,\ 1$ (Fig. 1).

However, the condition $M = const$ for changing a was Q unphysical, which is obvious from the CR mass formula (2)-(3). This led to a physically absurd result, when with growth a and Q the contour of the shadow shrinks and the area of the shadow decreases [7,8,9]. From a physical point of view, growth of a and Q increases the energy of the source and therefore should enhance the effects of gravity, increasing the area of the shadow contour.

In this letter, the same shadow contours are constructed using the KN metric with irreducible mass (3)-(4) and under a physically correct condition $M_{ir} = const$. This then led to the result opposite to [7,8,9], which, however, is natural from a physical point of view. In Fig. 2 are presented 9 contours with three values of each parameter $a, Q = 0,\ 1/2,\ 1$, which clearly show that growth of a and/or Q under physically natural condition $M_{ir} = 1$ leads to an increase in the area of the shadow contour.

As noted above and in [6], this particular result is physically correct, since growth of a and Q enhances the effects of gravity, including the shadow effect, which is manifested in an increase in its area and a change in the nature of deformation.

A more detailed description of the consequences of metrics with independent parameters and their further applications will be given in the article [1] and in the book [2].

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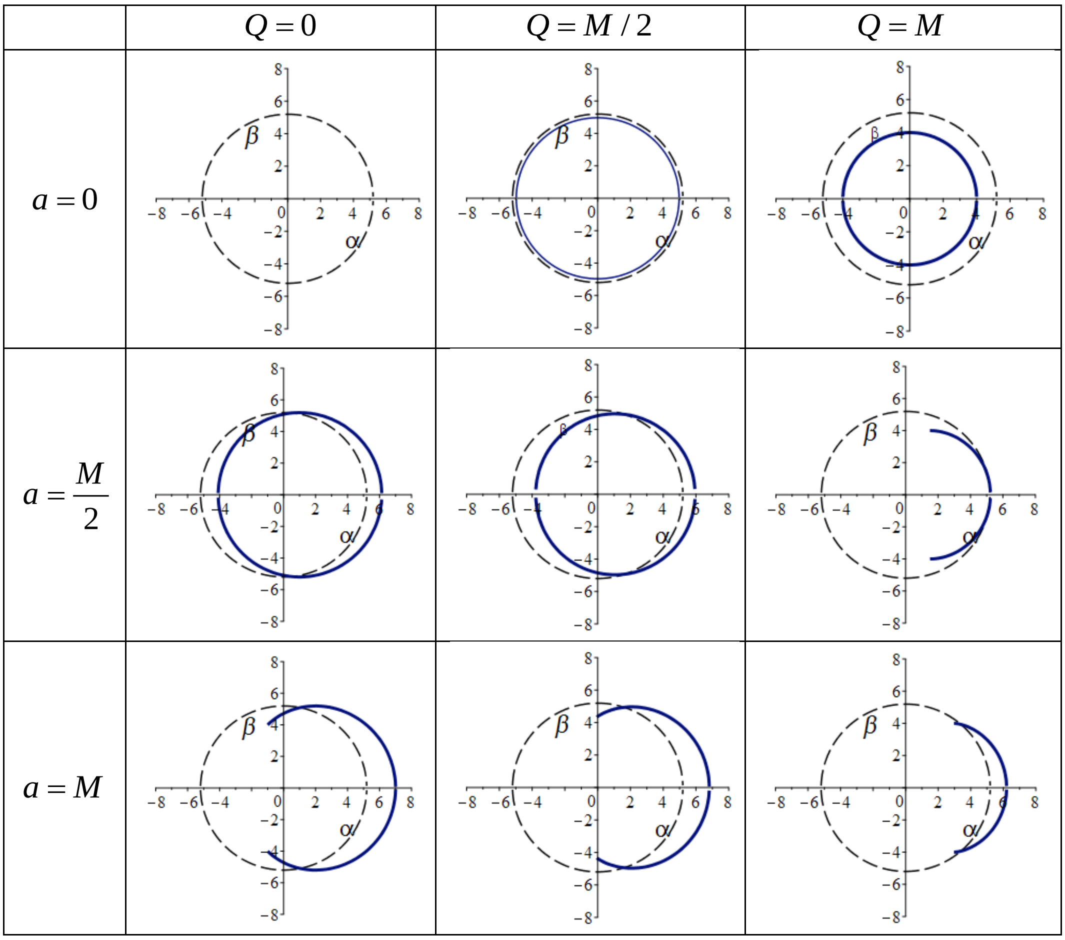

Fig. 1. The contour of the shadow of a compact source in the standard form of the Kerr-Newman metric (1) at the condition $M = const = 1$ in (6)-(7) ($\theta = \pi / 2$) (see [7]).

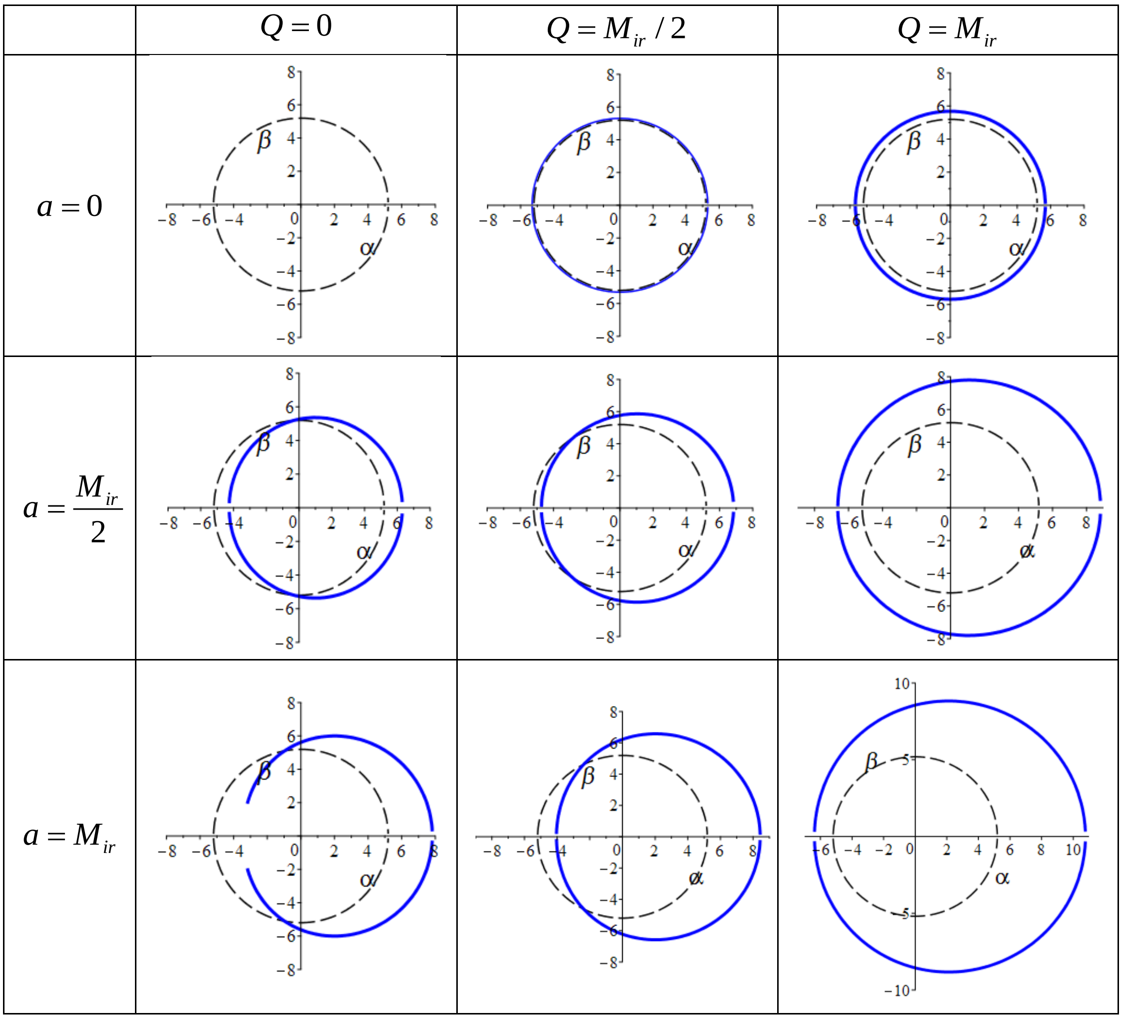

Fig. 2. Shadow contour of a compact source in the Kerr-Newman metric with irreducible mass (4), when (3) inserted into (6)-(7) and the condition $M_{ir} = const = 1$ is used ($\theta = \pi / 2$) (see [6]).