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Time-symmetric quantization of relativistic fields.
1. Complex fields, massless gauge fields and gravitons

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Abstract

In the standard formulation of quantum field theory (QFT), where there are only positive energy particles and antiparticles, the energy and charge of the vacuum diverge, which, due to the existence of gravity, leads to the inconsistency of the theory (cosmological constant problem). In the article, it is shown that in the Stueckelberg-Feynman (SF) interpretation, where antiparticles are described as negative energy particles moving backward in time, the zero-point energy and zero-point charge of vacuum of complex fields are absent and there is no cosmological constant problem. However, until now it was believed that the SF interpretation leads to negative probabilities and incompatible with QFT. In the article, it is presented a new formulation of QFT on the basis of the SF interpretation in the form of time-symmetric quantization (TSQ), where the probability of states is positive. In TSQ, the consequences of the SF interpretation are taken into account consecutively and it is shown that: a) the negative sign of the norm of states only changes the sign of the wave function, and not the probabilities; b) the expression of backward in time integrals through the forward in time integrals changes sign; c) the time ordering of the operators is symmetric in time and writing them through the usual ordering leads to the standard diagram technique. For this reason, TSQ correctly describes the known observable effects. In TSQ, the results of unification models change, in particular, a) for complex fields there is no zero-point energy even with broken supersymmetry; b) there is no zero-point energy of modes in string theories, which allows to include gravity, but there is no a conformal anomaly and the dimension of space can be arbitrary.

Keywords: quantization, zero-point energy, diagram technique, Standard Model, cosmological constant

Content

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1. Introduction

The covariant formulation of relativistic quantum theory contains particles of both positive and negative energies. The Stückelberg-Feynman (SF) interpretation [1,2] (see [3]), widely used in particle physics, is based on *the principle of equivalence* of positive energy antiparticles, going only forward in time, to negative energy particles going only backward in time. However, when fields are quantized, the negative energy states have a negative norm in Fock space, and it was believed that this leads to negative probabilities (see [4]). For this reason, the SF interpretation was considered as incompatible with quantum field theory (QFT).

As a result, the formulation of QFT with particles and antiparticles of positive energy, where vacuum is the lowest energy state, has become accepted as a standard one [5,6]. Such a description was introduced by manually replacing the creation (annihilation) operators of negative energy particles with the annihilation (creation) operators of positive energy antiparticles: $a^{+}_{-k} \to b_k$ ($a_{-k} \to b^{+}_{k}$). However, due to such a replacement, the fields had a divergent zero-point energy (and charged ones also had a zero-point charge), even when it did not exist at negative energies, in particular for complex fields.

At solving practical problems, the problem of vacuum energy was ignored. This was done by postulating a normal ordering recipe, which meant the rejection of divergent vacuum energy and charge of free fields. However, the existence of gravity does not allow one to ignore the vacuum energy, and this led to the inconsistency of the standard formulation of QFT. This fundamental problem, known as the cosmological constant problem, has also not been solved in all generalizations of the standard QFT and the Standard Model (SM) of particle physics [7].

At the same time, at returning to the covariant formulation of QFT with negative energies, the problem with the vacuum energy disappears in the particular case of complex fields. In the present article such an approach is developed and it is formulated *the time-symmetric quantization* (TSQ), in which the probabilities of states are positive and there is no contribution from complex fields to the cosmological constant. In TSQ, the SF interpretation is formulated more consistently than it was done before. In particular, it was taken into account that:

1) the negative sign of the norm of a state in Fock space changes only the sign of the wave function, while the probabilities remain positive;
2) a symmetrical time ordering of operators is necessary for mutually opposite directions of time evolution of states of different energy signs;
3) causal propagators arise naturally without manual rearrangement of field operators;
4) the expression of the backward in time integral in terms of the forward in time integral changes the sign of the integral.

It is shown in the article that TSQ leads to a logically consecutive and physically consistent formulation of QFT without zero-point energy and zero-point charge of vacuum of those relativistic fields that are described by complex fields[2]. In particular, transverse photons can be described as a complex scalar field with a chiral charge, the role of which plays helicity. The transverse quanta of massless gauge fields of SM and gravitons can be quantized similarly.

It is shown in the article that causal propagators and diagram technique can be constructed in TSQ in a natural way without former heuristic rules, and the final results coincide with results of standard diagram technique. Thus, TSQ correctly describes the known observable effects.

In TSQ, the results of unification models change, since there will be no zero-point energy even with broken supersymmetry between complex fields, and in a consistent theory of relativistic strings there will be no zero-point energy of modes, which allows to include gravity, but there will be no conformal anomaly, which does not give reasons for higher dimensions and fixing the dimension of space.

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[2] At revision 26.07.2023, typos and some formulations are corrected, the principle of equivalence is introduced and the sign of the probability flux of quanta is left negative when the flux is directed opposite to the time axis.

A description of TSQ and its application to free complex fields and massless fields of SM and gravitons are given in Section 2. Propagators, interacting fields, and diagram techniques are discussed in Sections 3-4. In the Appendix the applications of TSQ to complex harmonic oscillator and relativistic strings are presented. Applications of TSQ to theories with spontaneous symmetry breaking will be considered in forthcoming articles. A more detailed presentation of the TSQ will be given in the book [8].

2. Time-symmetric quantization of relativistic fields

2.1. SF interpretation of negative energy states

The standard formulation of QFT includes only positive energy states with Lagrangians $L_{+}^{(0)}(q_{+},\dot{q}_{+})$ and $L_{a+}^{(0)}(q_{a+},\dot{q}_{a+})$ for free particles and antiparticles, respectively, which move forward in time only. This is also expressed in time integrals, and the action function for them has the form:

$$S(t_1,t_0)=\int_{t_0}^{t_1} dt(L_{+}^{(0)}+L_{a+}^{(0)}), \quad t_1>t_0. \tag{1}$$

The covariant form of relativistic quantum theory is simpler and more natural when there are only particles of both signs of energy and no antiparticles. According to the SF interpretation, negative energy particles going only backward in time are equivalent to positive energy antiparticles going only forward in time. Therefore, further in the article, *time-symmetric quantization* (TSQ) is presented, based on the consistent implementation of SF interpretation in the theories of relativistic systems, in particular, relativistic fields.

In TSQ, the action function (1) takes the form:

$$S(t_1,t_0)=\int_{t_0}^{t_1} dtL_{+}^{(0)}+\int_{t_1}^{t_0} dtL_{-}^{(0)}, \quad t_1>t_0, \tag{2}$$

where $L_{-}^{(0)}$ is the free Lagrangian of a negative energy particle. Integration in the second term of Eq. (2) goes in the opposite direction to the time axis t, i.e. $L_{-}^{(0)}$ integrates from the initial moment t_1 to an earlier end point t_0.

However, the total Lagrangian $L=L^{(0)}+L_I$, unlike the free Lagrangians $L_{+}^{(0)}$ and $L_{-}^{(0)}$, is not diagonal, since it includes an interaction L_I mixing positive and negative energy states. Therefore, L can not be represented as a sum of terms, each of which refers to one sign of energy.

Thus, time integration must be made uniform for all terms of the total Lagrangian L and it is more convenient to integrate everywhere in the forward time direction. To do this, we interchange the limits in the second integral in (2), which gives a minus sign in front of L_{-}. As a result, the integration in the second term of (2) will be the same as in the first term with the same limits, and the action function with L can be written in the compact form by usual forward time integral:

$$S(t_1,t_0)=\int_{t_0}^{t_1} Ldt, \qquad L=L_{+}^{(0)}+(-L_{-}^{(0)})+L_I, \quad t_1>t_0. \tag{3}$$

Since $L_{-}^{(0)}$ is negative-definite and differs in sign from $L_{a+}^{(0)}$, then $-L_{-}^{(0)}$ is positive-definite and equal $L_{a+}^{(0)}$ with replacement $q_{a+},\dot{q}_{a+}$ by $q_{-},\dot{q}_{-}$.

The expression of the integrals directed opposite to the time axis t in terms of the integrals in the forward time direction, leading to a sign change, is also convenient for propagators of relativistic fields and will be considered in Section 3.1.

Notice that in TSQ, as the SF interpretation, the particle physics models and the experimental data are described directly without antiparticles. It is enough to know that the initial state of the negative energy particle $-p_{\mu}$ describes the final state of the antiparticle with $+p_{\mu}$. This is related also by the crossing symmetry, one of the basic symmetries of particle physics.

2.2. Complex scalar and vector fields

Energy k_0 of quanta of a relativistic scalar field ϕ is related to the momentum $\mathbf{k}$ by the relativistic relation $k_{0\pm}=\pm\omega_k$, $\omega_k=\sqrt{\mathbf{k}^2+m^2}$ and the complete set of solutions of the field equations includes states with both signs of energy. In the SF interpretation, the particles of negative energy k_{0-} going backward in time are equivalent to the antiparticles of energy k_{0+} moving forward in time.

For this reason, relativistic fields are generally described by complex field functions. Therefore, relativistic particles will be somewhat different from their antiparticles and are improbable to be truly neutral particles.

After writing the backward in time integrals in terms of forward in time integrals, the action function for a complex scalar field $\phi(\mathbf{x},t)$ and its Hermitian conjugate component $\phi^{+}(\mathbf{x},t)$ takes the compact form ($\mu=0,1,2,3$):

$$S=\int d^4x\,(\partial_{\mu}\phi^{+}\cdot\partial^{\mu}\phi-m^2\phi^{+}\phi)\ . \tag{4}$$

The corresponding Hamiltonian, charge operator, and canonical momenta are:

$$H=\int d^3x\,(\pi\pi^{+}+\nabla\phi^{+}\cdot\nabla\phi+m^2\phi^{+}\phi), \tag{5}$$

$$Q=i\int d^3x\,(\phi^{+}\pi^{+}-\pi\phi), \tag{6}$$

$$\pi(x)=\frac{\partial L}{\partial(\partial_t\phi)}=\partial_t\phi^{+},\ \ \pi^{+}(x)=\frac{\partial L}{\partial(\partial_t\phi^{+})}=\partial_t\phi. \tag{7}$$

The field equations and equal time commutators have the form:

$$(\partial_{\mu}\partial^{\mu}-m^2)\phi=0\ ,\quad (\partial_{\mu}\partial^{\mu}-m^2)\phi^{+}=0\ . \tag{8}$$

$$[\phi(\mathbf{x},t),\pi(\mathbf{x}',t)]=[\phi^{+}(\mathbf{x},t),\pi^{+}(\mathbf{x}',t)]=i\delta^3(\mathbf{x}-\mathbf{x}'). \tag{9}$$

Solutions of field equations (8) lead to the frequency expansion of field functions and canonical momenta:

$$\begin{aligned}&\phi(x)=\phi_{+}+\phi_{-}=\int d\tilde{k}(a_k e^{-ikx}+a_{-k}e^{ikx}),\quad \int d\tilde{k}\equiv\int\frac{d^2k}{(2\pi)^3 2\omega_k},\\ &\phi^{+}(x)=\phi_{+}^{+}+\phi_{-}^{+}=\int d\tilde{k}(a_k^{+}e^{ikx}+a_{-k}^{+}e^{-ikx}),\end{aligned} \tag{10}$$

$$\pi(x) = \pi_+ + \pi_- = i\int d\tilde{k}\omega_k (a_k^* e^{ikx} - a_{-k}^* e^{-ikx}),$$
$$\pi^+(x) = \pi_+^+ + \pi_-^+ = -i\int d\tilde{k}\omega_k (a_k e^{-ikx} - a_{-k} e^{ikx}). \quad (11)$$

Substituting (10)-(11) into (9), we obtain commutators for the creation-annihilation operators of field quanta of both signs of energy, non-zero of which are:

$$[a_k, a_{k'}^+] = 2\omega_k (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}'), \quad (12)$$

$$[a_{-k}, a_{-k'}^+] = -2\omega_k (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}'). \quad (13)$$

Consequences of the minus sign in (13) are discussed below in Section 2.3. The energy and charge of the field (5)-(6) are expressed in terms of the creation-annihilation operators as:

$$H = \int d\tilde{k}\, \omega_k (a_k^+ a_k + a_{-k}^+ a_{-k}), \quad (14)$$

$$Q = \int d\tilde{k} (a_k^+ a_k - a_{-k}^+ a_{-k}). \quad (15)$$

The annihilation operators define vacuum for both types of particles:

$$a_k |0\rangle = 0, \quad a_{-k} |0\rangle = 0, \quad (16)$$

which, as seen from (14)-(15), do not contain zero-point energy and zero-point charge. Single-particle states are defined as:

$$a_k^+ |0_+\rangle = |k\rangle, \quad a_{-k}^+ |0_-\rangle = |-k\rangle. \quad (17)$$

A massive complex vector field $B_\mu(x)$ contains the polarization vector $\varepsilon_{\mu k}^{\lambda}$ and its frequency expansion is [5,6]:

$$B_\mu(x) = \sum_\lambda \int d\tilde{k} (a_{k\lambda} \varepsilon_{\mu k}^{\lambda} e^{-ikx} + a_{-k\lambda} \varepsilon_{\mu k}^{\lambda +} e^{ikx}),$$
$$B_\mu^+(x) = \sum_\lambda \int d\tilde{k} (a_{k\lambda}^+ \varepsilon_{\mu k}^{\lambda +} e^{ikx} + a_{-k\lambda}^+ \varepsilon_{\mu k}^{\lambda} e^{-ikx}). \quad (18)$$

After separating three independent degrees of freedom, each of them is quantized in a given frame of reference independently as three complex scalar fields. Therefore, the Hamiltonian and the charge operator of the vector field have the form:

$$H = \sum_\lambda \int d\tilde{k}\, \omega_k (a_{k\lambda}^+ a_{k\lambda} + a_{-k\lambda}^+ a_{-k\lambda}),$$
$$Q = \sum_\lambda \int d\tilde{k} (a_{k\lambda}^+ a_{k\lambda} - a_{-k\lambda}^+ a_{-k\lambda}). \quad (19)$$

They also do not contain zero-point energy and zero-point charge.

This result about the absence of zero-point energy and zero-point charge of the vacuum of a complex vector field, obtained in one frame of reference, is valid for all frames of reference due to the invariance of the vacuum.

Thus, in TSQ the operators of observables of the complex scalar and vector fields are normal ordered automatically and there is no zero-point energy and zero-point charge.

2.3. Norm and probability of negative energy states

Let us consider the norm and the amplitude of probability of single-particle negative energy states, taking as an example, a complex scalar field. The negative sign of the commutator in (13) leads to the negative norm of such states in Fock space:

$$\langle -k' | -k \rangle = \langle 0_- | a_{-k'} a^+_{-k} | 0_- \rangle = -2\omega_k (2\pi)^3 \delta^3(\mathbf{k}-\mathbf{k}') \langle 0_- | 0_- \rangle. \tag{20}$$

This fact has often been interpreted as the appearance of a negative probability for such states. However, the wave function $\psi_{-k'}(x)$ (probability amplitudes) of this state, the square of whose modulus is the probabilities of the state (more precisely, the probability density, which we will not indicate further), is defined as a matrix element from the field operator (10) (see [5,6]), which using (20) gives for this probability amplitude:

$$\begin{aligned}\psi_{-k}(x) &= \langle 0_- | \phi(x) | -k \rangle = \int d\tilde{k}' \langle 0_- | a_{-k'} | -k \rangle e^{ik'x} = \\ &= \int d\tilde{k}' \langle 0_- | a_{-k'} a^+_{-k'} | 0_- \rangle e^{ik'x} = -e^{ikx}.\end{aligned} \tag{21}$$

Here the negative sign of the norm in (20) changes only the sign of the wave function of the state (21), while the probability, determined by the expression bilinear under the wave function, remains positive.

As a solution to the Klein-Gordon equation, the wave function (21) and the square of its modulus are scalars, not 4-components of a vector. Therefore, the probability density is defined in terms of the 4-vector of probability flux (current) S^μ:

$$S_{\mu\pm} = \frac{i}{2m} (\psi^+_\pm \overleftrightarrow{\partial}_\mu \psi_\pm). \tag{22}$$

The spatial components of this current $\mathbf{S}_\pm$ give currents of different signs along each axis of spatial coordinates, depending on the direction of the current relative to this axis:

$$\mathbf{S}_\pm = \frac{\mathbf{k}}{m} \psi^+_\pm \psi_\pm. \tag{23}$$

Similarly, the timelike components of the current $S_{0\pm}$ also have different signs depending on whether particles of two energy signs are evolve along or against the time axis:

$$S_{0\pm} = \pm \frac{\omega_k}{m} \psi^+_\pm \psi_\pm. \tag{24}$$

At normalized to $2\omega_k$ particles per unit volume $V = 1$ (on the hypersurface of simultaneity $t = const$), the sums of probability current of states for each of the energy signs are given by integrals (see [6]):

$$\int dV\, n_0 S^0_\pm = \pm \frac{\omega_k}{m}, \tag{25}$$

where $n_{0\pm} = \pm 1$ are timelike normals to this hypersurface. Here the minus sign refers to the negative-energy particles, the probability flux of which is directed to the past.

Thus, although the norm (20) of a single-particle state (as well as many-particle states with an odd number of particles) is negative, nevertheless, the probabilities of states are positive, even though the probability current is negative. In this case, the different sign of the probability fluxes for particles of different energy signs expresses only the fact that in the SMC these two types of fluxes are directed in opposite directions of the time axis.

2.4. Photon field

In the rest frame of the photon source, where the x^3 axis is directed along the photon momentum, the photon field has only two transverse physical components $(0, A_1, A_2, 0)$, which should be quantized. For this reason, the gauge $A_0 = A_3 = 0$ expresses the real physical situation in this case.

Nevertheless, another orientation of the axes $x^{3'}$ will lead to a non-zero component $A_{3'}$, expressed through A_1, A_2, and in other inertial reference frames, the Lorentz transformation will also generate a non-zero Coulomb component $A_{0'}$, also expressed through A_1, A_2. Therefore, in an arbitrary inertial frame, the photon field has all four components $A_{\mu'} = e^1_{\mu'} A_1 + e^2_{\mu'} A_2$ with 4-vector of polarization $\varepsilon_{\mu'} = e^1_{\mu'} \varepsilon_1 + e^2_{\mu'} \varepsilon_2$. Here the tetrads $(e^1_{\mu'}, e^2_{\mu'})$ are constant kinematic factors that perform the Lorentz transformation from the considered rest frame of the photon source to the given frame of reference.

For this reason, at considering the physical aspects, it is sufficient to quantize the physical components of the photon field in the rest frame of the source, and then the results can be transformed to the required frame of reference.

At the condition $\partial_\mu A^\mu = 0$ the action function for the photon field in terms of the usual time integrals takes the form:

$$S = -\frac{1}{4} \int d^4x \; F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} \int d^4x \; \partial_\mu A^\nu \cdot \partial^\mu A_\nu . \quad (26)$$

In the rest frame of the source $\mathbf{k} = (0, 0, k^3)$ and the gauge $A_3 = A_0 = 0$, the transverse physical components of circularly polarized photons form an analogue of the complex scalar field $A = (A_1 + iA_2)/\sqrt{2}$, $A^+ = (A_1 - iA_2)/\sqrt{2}$. Then Lagrangian (26) and Hamiltonian, therefore, are analogous to the case of a complex scalar field:

$$L = \int d^3x \, \partial_\mu A^+ \cdot \partial^\mu A, \quad (27)$$

$$H = \int d^3x \, (\pi \pi^+ + \nabla A^+ \cdot \nabla A), \quad (28)$$

where $\pi(x) = \partial_t A^+$, $\pi^+(x) = \partial_t A$. The description of the field of circularly polarized photons in terms of complex variables diagonalizes the Hamiltonian (28) and helicity:

$$\Lambda = i \int d^3x \, (A^+ \pi^+ - \pi A). \quad (29)$$

The latter is, in fact, a chiral charge, and photons with helicities $\Lambda = \pm 1$ are like a particle and a charge-conjugate antiparticle.

The field equations $\partial_\mu \partial^\mu A = 0$, $\partial_\mu \partial^\mu A^+ = 0$ give the frequency expansion:

$$A = \int d\tilde{k} (a_k e^{-ikx} + a_{-k} e^{ikx}), \;\; \pi = i \int d\tilde{k} \omega_k (a^+_k e^{ikx} - a^+_{-k} e^{-ikx}), \quad (30)$$

where $a_{\pm k}^{+}, a_{\pm k}$ are the creation-annihilation operators for $\Lambda_{+} = +1$. For commutators, this gives:

$$[A(\mathbf{x},t),\pi(\mathbf{x}',t)] = i\delta^3(\mathbf{x}-\mathbf{x}'), \tag{31}$$

$$[a_{\pm k}, a_{\pm k'}^{+}] = \pm(2\pi)^3 2\omega_k \delta^3(\mathbf{k}-\mathbf{k}'), \tag{32}$$

At an arbitrary orientation of the coordinate axes, a projection operator will appear here, selecting the transverse components.

As shown in the previous Section 2.3, the negative sign of the commutator in (32) does not change the sign of the probability of states and they remain positive. Operators of observables are:

$$\begin{aligned} H &= \int d\tilde{k}\,\omega_k (a_k^{+} a_k + a_{-k}^{+} a_{-k}), \\ \Lambda^0 &= \int d\tilde{k} (a_k^{+} a_k - a_{-k}^{+} a_{-k}). \end{aligned} \tag{33}$$

This shows that the vacuums, defined as $a_{\pm k}|0\rangle = 0$, do not contain zero-point energy and zero-point helicity.

Due to the invariance of the vacuum, the vanishing of its energy in one of the frames of reference and in one of the gauges is true for all frames of reference and gauges.

2.5. Fermion field

In the theory of spin ½ fermions, the Dirac field ψ includes positive and negative energy spinors $\psi_{+}^{\alpha} \sim u_p^{\alpha}$ and $\psi_{-}^{\alpha} \sim v_p^{\alpha}$ for positive and negative energy states, respectively.

For this reason, it was completely unnatural for such a theory to introduce into it by “hands” an alien element - creation-annihilation operators for positive energy antiparticles, while retaining the spinors for the negative (!) energy states, as it was done in the standard formulation of QFT. This then required new manual semi-phenomenological operations. These include, in particular, normal ordering and the manual construction of propagators, while retaining the poles of the latter at negative energies.

The SF interpretation, on the contrary, is natural for the theory of fermions and it is not surprising that it arose there. In TSQ, for this reason, the theory of fermions, written in terms of ordinary time integrals, practically does not require serious modification. On the contrary, it is precisely in the TSQ, as will be seen below, the theory of fermions becomes consistent, without alien elements and semi-phenomenological recipes.

In the TSQ, the action function for the Dirac field with usual time integrals has the form:

$$S = \int d^4x \left(\frac{i}{2} [\bar{\psi}\gamma^{\mu}(\partial_{\mu}\psi) - (\partial_{\mu}\bar{\psi})\gamma^{\mu}\psi] - m\bar{\psi}\psi \right). \tag{34}$$

This gives the Hamiltonian, the charge operator, and the Dirac equation:

$$\begin{aligned} H &= \frac{i}{2}\int d^3x\, [\psi^{+}(\partial_t\psi) - (\partial_t\psi^{+})\psi], \\ Q &= \int d^3x\, \psi^{+}\psi, \end{aligned} \tag{35}$$

$$\gamma^{\mu} i\partial_{\mu}\psi - m\psi = 0. \tag{36}$$

The frequency decomposition of the field operators in terms of solutions of (36) gives:

$$\psi(x)=\psi_+ +\psi_- =2m\sum_\alpha\int d\tilde{p}(b_{p\alpha}u_p^\alpha e^{-ipx}+b_{-p\alpha}v_p^\alpha e^{ipx}),$$
$$\psi^+(x)=\psi_+^+ +\psi_-^+ =2m\sum_\alpha\int d\tilde{p}(b_{p\alpha}^+u_p^{\alpha+} e^{ipx}+b_{-p\alpha}^+v_p^{\alpha+} e^{-ipx}). \quad (37)$$

The bilinear combinations of spinors have the form ($E_p=(\mathbf{p}^2+m^2)^{1/2}$):

$$u_p^{\alpha+}u_p^{\alpha'}=v_p^{\alpha+}v_p^{\alpha'}=\frac{E_p}{m}\delta^{\alpha\alpha'}, \quad (38)$$

$$\bar{u}_p^\alpha u_p^{\alpha'}=-\bar{v}_p^\alpha v_p^{\alpha'}=\delta^{\alpha\alpha'}. \quad (39)$$

$$\sum_\alpha u_p^\alpha\bar{u}_p^\alpha=\frac{\gamma p+m}{2m},\qquad -\sum_\alpha v_p^\alpha\bar{v}_p^\alpha=\frac{-\gamma p+m}{2m}. \quad (40)$$

Since the wave functions of fermions are antisymmetric, the field operators must change sign at rearrangement. Therefore, when field operators are rearranged in commutators, their product changes sign and the commutator turns into an anticommutator. As a result, the fermion field is quantized by simultaneous anticommutators ($\pi=i\psi^+$):

$$\left\{\psi^\alpha(\mathbf{x},t),\pi^{+\alpha'}(\mathbf{x}',t)\right\}=i\left\{\psi^\alpha(\mathbf{x},t),\psi^{+\alpha'}(\mathbf{x}',t)\right\}=i\delta^3(\mathbf{x}-\mathbf{x}')\delta^{\alpha\alpha'}, \quad (41)$$

$$\left\{b_{\pm p\alpha},b^+_{\pm p'\alpha'}\right\}=\frac{E_p}{m}\delta^3(\mathbf{p}-\mathbf{p}')\delta_{\alpha\alpha'}. \quad (42)$$

As the result, the expressions (35)-(42) give for observables:

$$H=\sum_\alpha\int d\tilde{p}\frac{m}{E_p}(b^+_{p\alpha}b_{p\alpha}-b^+_{-p\alpha}b_{-p\alpha})E_p,$$
$$Q=\sum_\alpha\int d\tilde{p}\frac{m}{E_p}(b^+_{p\alpha}b_{p\alpha}+b^+_{-p\alpha}b_{-p\alpha}). \quad (43)$$

Thus, the fermionic ground states, defined as $b_{\pm p\alpha}\left|0\right\rangle=0$, also do not contain zero-point energy and zero-point charge. In SVR, zero energies of complex fields of fermions and bosons are absent, and if in supersymmetric theories with supersymmetry is weakly broken, then this does not lead to the appearance of zero vacuum energy.

Notice, that there is no direct analogy between the relations of the theories of fermions and bosons. If the properties of the quantized bosonic field are mainly determined by the creation-annihilation operators, then in fermion theory the properties of field operators are determined by a non-trivial combination of the properties of creation-annihilation operators, two kinds of 4-spinors u_p^α, v_p^α, together with the Dirac matrices γ^μ. In particular, the negative sign appears not for the probability current $v_p^{\alpha+}v_p^{\alpha'}$ in (38), but for the bilinear form $v_p^{\alpha+}\gamma^0 v_p^{\alpha'}=-\delta^{\alpha\alpha'}$ in (39) and the projection operator in (40). In addition, the anticommutator in $\{b_{-p\alpha},b^+_{-p'\alpha'}\}$ (42) also remains positive.

These unusual properties of fermionic states continue the series that was known earlier - that the canonical momentum has the form $\pi=i\psi^+$, and the eigenvalues of the velocity operator are equal to the speed of light. A more detailed discussion of these issues, including the possibility of modifying the formalism, will be given in subsequent publications.

2.6. Massless gauge fields and the field of gravitons

In the rest frame of the source, when the axis x^3 is directed along the momentum of the emitted quantum, the quanta of the massless vector gauge field A_μ^a (spin 1) and gravitons (spin 2) have two transverse physical states. There is axial symmetry here, and for circular polarization the free Hamiltonian and helicity are diagonal.

For this reason, their quantization is similar to the case of a photon field. Further consideration of nonlinearity, i.e. interactions of different field components, and internal symmetries does not change the conclusion about the absence of the zero-point energy of vacuum, which is part of the free Hamiltonian. The nonlinear contributions to the field energy are proportional to the coupling constants and decrease in the weak field limit, while the zero-point energy, as the energy $\omega_k / 2$ of vacuum fluctuations each of the physical components in the free Hamiltonian, does not depend on the interaction constants.

The conclusion that when massless non-Abelian fields and the graviton field are quantized in one frame of reference, zero-point energy does not arise in the transverse gauge, remains valid for all frames of reference and gauges due to the invariance of the vacuum [8].

3. Commutators, time ordering, and propagators

3.1. Commutators and symmetrical time ordering of operators

The commutator of the components of a complex scalar field ϕ is:

$$\begin{aligned}&[\phi(x'),\phi^*(x)] = \\ &= \int d\tilde{k}\, d\tilde{k}'\{[a_k, a^*_{k'}]e^{-i(\omega_{k'}t'-\omega_k t)} + [a_{-k}, a^*_{-k'}]e^{i(\omega_{k'}t'-\omega_k t)})e^{i(\mathbf{k}'\mathbf{x}'-\mathbf{kx})} = \\ &= \int d\tilde{k}\,(e^{-i\omega_k(t'-t)} - e^{i\omega_k(t'-t)})e^{i\mathbf{k}(\mathbf{x}'-\mathbf{x})} = D(x'-x),\end{aligned} \tag{44}$$

where $D(x'-x)$ is the Pauli-Jordan function, which disappears behind the light cone. For canonically conjugate field operators we have:

$$[\phi(x'),\pi(x)] = i\partial_t D(x'-x). \tag{45}$$

For a fermionic field, the canonical momentum is $\pi = i\psi^+$, and the analogue of (45) is:

$$\{\psi(x'),\bar{\psi}(x)\} = -(i\gamma^\mu\partial_\mu + m)iD(x'-x). \tag{46}$$

It is consistent with the equal time anticommutators (42).

The standard time ordering operator T_+ acts on the positive energy state operators only and arranges them in order of increasing time from right to left:

$$T_+[A_+(t')B_+(t)] = \begin{cases} A_+(t')B_+(t), & t' > t, \\ B_+(t)A_+(t'), & t' < t. \end{cases} \tag{47}$$

In TSQ there is also the inverse to T_+ operator T_-, which acts only on the operators of negative energy states and orders them in reverse order as time decreases from right to left:

$$T_-[A_-(t')B_-(t)] = \begin{cases} B_-(t)A_-(t'), & t' > t, \\ A_-(t')B_-(t), & t' < t. \end{cases} \tag{48}$$

In the general case, the symmetric time ordering operator $\hat{T}$ is introduced as the product of the previous two ones $\hat{T} = T_+ T_-$. It has the properties $\hat{T}A_+ = T_+ A_+$, $\hat{T}A_- = T_- A_-$ and acts selectively on the operators of both signs of energy, ordering them in reverse order. In particular, when $t' > t$ we have:

$$\hat{T}[A_+(t')B_+(t)A_-(t')B_-(t)] = [A_+(t')B_+(t)][B_-(t)A_-(t')]. \qquad (49)$$

3.2. Causal propagators of fields

In TSQ, the SF causal propagators $G_c(x - x')$ appear naturally without manual rearrangement of field operators, as was done in the standard formulation of QFT. For a scalar field, G_c is a Green's function under the SF boundary conditions corresponding to the propagation forward in time of positive energy and backward the negative energy ones.

For the Klein-Gordon equation with the source $j(x)$:

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = j(x) \qquad (50)$$

propagators $G_{c\pm}(x' - x)$ for solutions of two energy signs are defined as:

$$(\partial_\mu \partial^\mu + m^2)G_{c\pm}(x' - x) = \delta^4(x' - x). \qquad (51)$$

The corresponding solution of the field equations (50), taking into account the translational invariance and the SF boundary conditions, has the form:

$$\phi(x') = \phi^{(0)}(x') + \int_{-\infty}^{\infty} dt \int d^3x\, G_{c+}(x' - x)\, j(x) + \int_{\infty}^{-\infty} dt \int d^3x\, G_{c-}(x' - x)\, j(x). \qquad (52)$$

Here $\phi^{(0)}$ is the solution of a homogeneous equation with the same boundary conditions, $G_{c+}(x' - x) \sim \theta(t' - t)$ describes the propagation of positive energy forward in time, and - $G_{c-}(x' - x) \sim \theta(t - t')$ the propagation of negative energy backward in time.

The limits of the second time integral in (52) are reversed (going from the future to the past) and therefore the integrals of these two terms cannot simply be combined, so that the full causal propagator G_c cannot be represented as the sum of G_{c+} and G_{c-}. However, we can flip the limits of the second integral so that it becomes the same as the first integral. This gives a minus sign in front of G_{c-}, and (52) takes a more compact form:

$$\phi(x) = \phi^{(0)}(x) + \int_{-\infty}^{\infty} dt' \int d^3x' \,[G_{c+}(x - x') - G_{c-}(x - x')]\, j(x'), \qquad (53)$$

or

$$\phi(x) = \phi^{(0)}(x) + \int_{-\infty}^{\infty} dt' \int d^3x'\, G_c(x - x')\, j(x'), \qquad (54)$$

$$G_c(x) = G_{c+}(x) - G_{c-}(x), \qquad (55)$$

Thus, at expressing the backward in time integral in terms of the forward in time integral, the SF causal propagator G_c is given by the *difference* of G_{c+} and G_{c-}.

Causal propagators $G_{c\pm}$ for a complex scalar field are defined as mutually inverse time ordered products of field operators for two signs of energy:

$$iD_{c\pm}(x'-x)=\langle 0|T_{\pm}[\phi_{+}(x')\phi_{\pm}^{*}(x)]|0\rangle. \tag{56}$$

Substitution of frequency expansions gives (the calculations are presented in detail in order to clearly show the differences with the standard approach):

$$\begin{aligned}
iD_{c+}(x'-x)&=\langle 0|T_{+}[\phi_{+}(x')\phi_{+}^{*}(x)]|0\rangle=\\
&=\langle 0|\phi_{+}(x')\phi_{+}^{*}(x)|0\rangle\theta(t'-t)+\langle 0|\phi_{+}^{*}(x)\phi_{+}(x')|0\rangle\theta(t-t')=\\
&=\int d\tilde{k}d\tilde{k}'[\langle 0|a_{k'}a_{k}^{*}|0\rangle\theta(t'-t)e^{-i(k'x'-kx)}+\langle 0|a_{k}^{*}a_{k'}|0\rangle\theta(t-t')e^{i(k'x'-kx)}]=\\
&=\int d\tilde{k}\,\theta(t'-t)e^{-ik(x'-x)}.
\end{aligned} \tag{57}$$

$$\begin{aligned}
iD_{c-}(x'-x)&=\langle 0|T_{-}[\phi_{-}(x')\phi_{-}^{*}(x)]|0\rangle=\\
&=\langle 0|\phi_{-}^{*}(x)\phi_{-}(x')|0\rangle\theta(t'-t)+\langle 0|\phi_{-}(x')\phi_{-}^{*}(x)|0\rangle\theta(t-t')=\\
&=\int d\tilde{k}d\tilde{k}'[\langle 0|a_{-k}^{*}a_{-k'}|0\rangle\theta(t'-t)e^{-i(k'x'-kx)}+\langle 0|a_{-k'}a_{-k}^{*}|0\rangle\theta(t-t')e^{i(k'x'-kx)}]=\\
&=-\int d\tilde{k}\,\theta(t-t')e^{ik(x'-x)}.
\end{aligned} \tag{58}$$

Since further it will be convenient to transform the backward in time integrals into the ordinary time integrals, which changes sign, it is convenient not to indicate this sign change explicitly, but to combine it with the time ordering operator, defining that the operator $\hat{T}$ changes sign of T_{-} also. Thus, the total causal propagator in (54) - (55) takes the standard form:

$$\begin{aligned}
iD_{c}(x'-x)&=\langle 0|\hat{T}\phi(x')\phi^{*}(x)|0\rangle=\\
&=\langle 0|\phi_{+}(x')\phi_{+}^{*}(x)|0\rangle\theta(t'-t)-\langle 0|\phi_{-}(x')\phi_{-}^{*}(x)|0\rangle\theta(t-t')=\\
&=\int d\tilde{k}[\theta(t'-t)e^{-ik(x'-x)}+\theta(t-t')e^{ik(x'-x)}].
\end{aligned} \tag{59}$$

The standard result also leads to the standard momentum representation:

$$iD_{c}(x'-x)=\frac{1}{(2\pi)^{4}}\int d^{k}k\,\frac{e^{-ik(x'-x)}}{k^{2}-m^{2}+i\varepsilon}. \tag{60}$$

Causal propagators for fermions $S_{c\pm}(x)$ can be obtained similarly, considering the time ordered products of field operators in two directions of time:

$$iS_{c\pm}(x'-x)=\langle 0|T_{\pm}[\psi_{\pm}(x')\bar{\psi}_{\pm}(x)]|0\rangle. \tag{61}$$

The calculations are also given in detail:

$$\begin{aligned}
iS_{c+}(x'-x)&=\langle 0|T_{+}[\psi_{+}(x')\bar{\psi}_{+}(x)]|0\rangle=\\
&=\langle 0|\psi_{+}(x')\bar{\psi}_{+}(x)|0\rangle\theta(t'-t)-\langle 0|\bar{\psi}_{+}(x)\psi_{+}(x')|0\rangle\theta(t-t')=\\
&=(2m)^{2}\sum_{\alpha'\alpha}\int d\tilde{p}d\tilde{p}'[\theta(t'-t)\langle 0|b_{p'\alpha'}b_{p\alpha}^{+}|0\rangle u_{p'}^{\alpha'}\bar{u}_{p}^{\alpha}e^{-i(p'x'-px)}-\\
&-\theta(t-t')\langle 0|b_{p\alpha}^{+}b_{p'\alpha'}|0\rangle v_{p'}^{\alpha'}\bar{v}_{p}^{\alpha}e^{i(p'x'-px)})]=\\
&=\sum_{\alpha}\int d\tilde{p}\,\theta(t'-t)(\gamma p+m)e^{-ip(x'-x)},
\end{aligned} \tag{62}$$

$$\begin{aligned}
&iS_{c-}(x'-x)=\langle 0|T_-[\psi_-(x')\bar{\psi}_-(x)]|0\rangle = \\
&=-\langle 0|\bar{\psi}_-(x)\psi_-(x')|0\rangle\theta(t'-t)+\langle 0|\psi_-(x')\bar{\psi}_-(x)|0\rangle\theta(t-t')= \\
&=(2m)^2\sum_{\alpha'\alpha}\int d\tilde{p}d\tilde{p}'[-\theta(t'-t)\langle 0|b^+_{p\alpha}b_{p'\alpha'}|0\rangle\bar{u}^{\alpha}_{p}u^{\alpha'}_{p'}e^{-i(p'x'-px)} - \\
&+\theta(t-t')\langle 0|b_{-p'\alpha'}b^+_{-p\alpha}|0\rangle v^{\alpha'}_{p'}\bar{v}^{\alpha}_{p}e^{i(p'x'-px)})]= \\
&=-\sum_{\alpha}\int d\tilde{p}\,\theta(t-t')(-\gamma p+m)e^{ip(x'-x)}.
\end{aligned} \tag{63}$$

In fermionic analogs of Eqs. (51)-(55), representing the backward in time integral through the forward in time integral adds a sign in front of $S_{c-}(x)$. Therefore, the full propagator $S_c(x)$ finally also coincides with the standard one:

$$\begin{aligned}
&iS_c(x'-x)=iS_{c+}(x'-x)-iS_{c-}(x'-x)=\langle 0|\hat{T}\psi(x')\bar{\psi}(x)|0\rangle = \\
&=\langle 0|\psi_+(x')\bar{\psi}_+(x)\theta(t'-t)-\psi_-(x')\bar{\psi}_-(x)\theta(t-t')|0\rangle = \\
&=\int d\tilde{p}\sum_{p}\Big[\theta(t'-t)(\gamma p+m)e^{-i\omega_p(t'-t)}+\theta(t-t')(-\gamma p+m)e^{i\omega_p(t'-t)}\Big]e^{i\mathbf{p}(\mathbf{x}'-\mathbf{x})} \\
&=-(i\gamma\partial+m)D_c(x'-x).
\end{aligned} \tag{64}$$

Thus, in TSQ, the symmetric time ordering operator $\hat{T}$ (taking into account the change in sign of T_- due to the joining together of time integrations) replaces the previous "rules" for constructing causal propagators, which consisted of manually rearranging the field operators to obtain the desired result.

4. Interacting fields and diagram technique in TSQ

4.1. Interaction representation and perturbation theory

In TSQ, two types of contributions to the free Hamiltonian are additive $H_0=H_{0+}+H_{0-}$ and the time dependence of the free field φ has the form $\varphi_\pm(t)=e^{\mp iH_{0\pm}t}\varphi_\pm(0)e^{\pm iH_{0\pm}t}$.

The time evolution of the interacting fields is described using the interaction Hamiltonian H_I. When the contribution of interactions with positive and negative energies is additive $H_I=H_{I+}+H_{I-}$, the time evolution is given by the evolution operator:

$$U(t,t_0)=\hat{T}\exp\left(-i\int_{t_0}^{t}dt'[\theta(t-t_0)H_{I+}(t')+\theta(t_0-t)H_{I-}(t')\right), \tag{65}$$

where $\hat{T}=T_+T_-$ is the symmetric time ordering operator defined in (49).

In the general case, there is a mixed part of the interaction $H_{I+-}=\tilde{H}_{I+}\tilde{H}_{I-}$, leading, in particular, to the creation and annihilation of pairs, and

$$H_I=H_{I+}+H_{I-}+\tilde{H}_{I+}\tilde{H}_{I-}. \tag{66}$$

For this reason, the backward in time integrals in (65) must be written in terms of the forward in time integrals, as was done for the free Lagrangian in Section 2.1, leading to sign changes in propagators, as was done in Section 3.2. By these explanations, we can write (65) in a more compact form:

$$U(t,t_0) = \hat{T}\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right). \tag{67}$$

In perturbation theory, the S - operator, defined as $\hat{S} = U(\infty,-\infty)$, takes the form:

$$\hat{S} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\int_{-\infty}^{\infty} dt_1 ... \int_{-\infty}^{\infty} dt_n \hat{T}[H_I(t_1)...H_I(t_n)]. \tag{68}$$

4.2. Diagram technique

For a two-point function, the modified Wick theorem takes the form:

$$\begin{aligned}\hat{T}[\phi(x')\phi^*(x)] &=: \phi(x')\phi^*(x): + \langle 0|\hat{T}[\phi(x')\phi^*(x)]|0\rangle = \\ &=: \phi(x')\phi^*(x): + iD_c(x'-x),\end{aligned} \tag{69}$$

where $:A:$ is normal ordering, and similarly for fermions. For example, in the electron-positron loop diagram, the product $\hat{T}[H_I(x')H_I(x)]$ gives the standard formulas:

$$\begin{aligned}\hat{T}[H_I(x')H_I(x)] &= T_+[\psi_{j+}(x')\bar{\psi}_{m+}(x)]\gamma_{\mu mn}T_-[\psi_{n-}(x)\bar{\psi}_{i-}(x')]\gamma^{\mu}_{ij} = \\ &= -S_{c+}(x'-x)_{jm}\gamma_{\mu mn}S_{c-}(x-x')_{ni}\gamma^{\mu}_{ij}, \qquad t' > t,\end{aligned} \tag{70}$$

$$\begin{aligned}\hat{T}[H_I(x')H_I(x)] &= T_-[\psi_{j-}(x')\bar{\psi}_{m-}(x)]\gamma_{\mu mn}T_+[\psi_{n+}(x)\bar{\psi}_{i+}(x')]\gamma^{\mu}_{ij} = \\ &= -S_{c-}(x'-x)_{jm}\gamma_{\mu mn}S_{c+}(x-x')_{ni}\gamma^{\mu}_{ij}, \qquad t' < t.\end{aligned} \tag{71}$$

Thus, there are following differences between TSQ and the standard diagram technique:

- instead of the operators of antiparticles, the operators of negative energy particles appear, and time integration for them goes into the past;
- the time ordering depends on the sign of the energy and is symmetrical in time.

These differences were effectively taken into account in the standard diagram technique by introducing some formal rules (normal ordering, constructing propagators, etc.) [5,6]. This explains the fact that the previous results basically coincide with the results of TSQ.

5. Conclusion

When relativistic fields with negative energy states are quantized, the norms of states are negative, but this only changes the sign of the wave function, and the probabilities of states, as bilinear combinations of the wave functions, remain positive.

In TSQ, formulated in the article and based on the SF interpretation, the time integration in the action function and in the perturbation theory goes in both directions of time, depending on the sign of the energy. Writing backward in time integrals through forward in time integrals, making formulas more compact, leads to a sign change.

It is shown that QFT with particles of both signs of energy moving in mutually opposite directions of time is equivalent to the theory with positive energy antiparticles, but does not contain zero-point energy and zero-point charge of vacuum. The absence of the contribution of free fields to the vacuum energy basically solves the cosmological constant problem.

In TSQ, the symmetric time ordering operator $\hat{T} = T_+ T_-$ is introduced, which directly leads to the SF causal propagators without manual rearrangement of operators. As the result, TSQ modifies the diagram technique for interacting fields, taking into account above circumstances, but retains the final results of the standard diagram technique and thus correctly describes the known observational effects.

Thus, TSQ provides the SF interpretation in QFT consecutively and it can be considered as a step towards a consistent and finite QFT.

Applications of TSQ to gauge theories with spontaneous symmetry breaking and mass generation mechanism will be considered in the second article and in the book [8].

Appendix.

A. Harmonic oscillator with complex coordinates and negative energy states

Consider a harmonic oscillator with complex coordinates q and q^*, which has negative energy states. The Lagrangian and Hamiltonian have the form:

$$L = m(\dot{q}^* \dot{q} - \omega^2 q^* q), \quad H = \frac{1}{m}(pp^* + m^2\omega^2 q^* q), \tag{72}$$

where $p = \partial L / \partial \dot{q} = m\dot{q}^*$, $p^* = \partial L / \partial \dot{q}^* = m\dot{q}$. Canonical Equations $\dot{p} = -\partial H / \partial q$, $\dot{p}^* = -\partial H / \partial q^*$ give the equations of motion $\ddot{q} + \omega^2 q = 0$, $\ddot{q}^* + \omega^2 q^* = 0$ and their solutions are the sum of the contributions of positive and negative energies:

$$\begin{aligned} q &= q_+ + q_- = (2m\omega)^{-1}(a_+ e^{-i\omega t} + a_- e^{i\omega t}), \\ q^* &= q_+^* + q_-^* = (2m\omega)^{-1}(a_+^* e^{i\omega t} + a_-^* e^{-i\omega t}). \end{aligned} \tag{73}$$

The corresponding momenta have the form:

$$\begin{aligned} p &= p_+ + p_- = (i/2)(a_+^* e^{i\omega t} - a_-^* e^{-i\omega t}), \\ p^* &= p_+^* + p_-^* = -(i/2)(a_+ e^{-i\omega t} - a_- e^{i\omega t}). \end{aligned} \tag{74}$$

The creation-annihilation operators are expressed in terms of canonical variables as:

$$a_\pm = (m\omega q_\pm \pm i p_\pm^*) e^{\pm i\omega t}, \quad a_\pm^* = (m\omega q_\pm^* \mp i p_\pm) e^{\mp i\omega t}. \tag{75}$$

Quantization gives commutators for canonical variables:

$$\begin{aligned} &[q, p] = (qp - pq) = \frac{i}{4m\omega}\left([a_+, a_+^*] - [a_-, a_-^*]\right) = i, \\ &[q^*, p^*] = i, \quad [q, q^*] = [p, p^*] = 0. \end{aligned} \tag{76}$$

In this case, the commutators of creation-annihilation operators are equal to:

$$[a_+, a_+^*] = 2m\omega, \quad [a_-, a_-^*] = -2m\omega. \tag{77}$$

The ground states are defined as $a_\pm |0_\pm\rangle = 0$, and the excited states as:

$$a_\pm^* |n_\pm\rangle = \sqrt{n_\pm + 1}\,|n_\pm + 1\rangle, \quad a_\pm |n_\pm\rangle = \sqrt{n_\pm}\,|n_\pm - 1\rangle, \quad n_\pm = 0, 1, \ldots \tag{78}$$

The total Hamiltonian H and the number operator of quanta N have the form:

$$H = H_+ + H_- = \omega(a_+^* a_+ + a_-^* a_-) / 2m\omega,$$
$$N = N_+ + N_- = (a_+^* a_+ - a_-^* a_-) / 2m\omega. \quad (79)$$

As we can see, the energy levels are equidistant, and there is no zero energy in the ground state.

This corresponds to the spectrum of purely rotational levels of a planar oscillator (i.e. a harmonic rotator), where there are two modes with opposite directions of rotation (quanta and anti-quanta), and the energy of the rotational modes is proportional to the angular momentum $E_{n\pm} = |J_{n\pm}|\omega$, which is quantized $J_{n\pm} = n_\pm \hbar$. Anti-quanta are equivalent to the negative energy quanta evolving backward in time. The lack of the zero-point energy in (79) does not change at passing to anti-quanta of positive energy, since $|\langle 0_- | H_- | 0_- \rangle| = \langle 0_{a+} | H_{a+} | 0_{a+} \rangle$.

Thus, the theory of a complex harmonic oscillator containing negative energy states is consistent and compatible with the SF interpretation, moreover the Lagrangian (72) leads to the Hamiltonian (79) without zero-point energy.

Notice that although the negative sign of the second commutator in (77) leads to the negative norm of negative energy states, however, as was shown in Section 2.2 for a scalar field, this only changes the sign of the wave function, defined as a matrix element from the canonical coordinate (21), and the probability of states, as a bilinear combination of wave functions, remains positive.

B. Consistent string theory without conformal anomaly

Theories of relativistic string are based on the fact that, at quantization, they lead to a critical dimension of spacetime: $D = 26$ for bosonic and $D = 10$ for fermionic strings (including superstrings). In fact, the critical dimension is a consequence of the presence of divergent zero-point energies of string modes (see details in [8]). It arises from the requirement to eliminate anomalies arising from the residual contribution of the zero-point energy of string modes after their “regularization”.

Consider the application of TSQ to strings. For a bosonic string, the action function is:

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\partial_\alpha X_\mu \partial_\beta X^\mu = -T\int_{\tau_0}^{\tau_1} d\tau \int d\sigma\, \partial_\alpha Y_\mu^* \cdot \partial^\alpha Y^\mu, \quad (80)$$

where $Y^\mu = (X_R^\mu + iX_L^\mu)/\sqrt{2}$ and $Y^{\mu *} = (X_R^\mu - iX_L^\mu)/\sqrt{2}$ are two positive energy modes moving in opposite directions. In TSQ, one of the modes is chosen as the basic one, for example, the right-hand mode with positive energy $Y_+^\mu(\tau - \sigma)$, moving forward in time, and the left-hand positive-energy mode $Y_+^\mu(\tau + \sigma)$ can then be described as the same right-handed one, but with negative energy and moving backward in time.

For closed strings, the frequency modes for complex coordinates take the form:

$$Y^\mu(\tau - \sigma) = Y_0^\mu + \frac{il}{2}\sum_{n=1}^{\infty}\frac{1}{n}(\alpha_n^\mu e^{-2in(\tau-\sigma)} + \alpha_{-n}^\mu e^{2in(\tau-\sigma)}),$$
$$Y^{\mu *}(\tau - \sigma) = Y_0^{\mu *} - \frac{il}{2}\sum_{n=1}^{\infty}\frac{1}{n}(\alpha_n^{\mu *} e^{2in(\tau-\sigma)} + \alpha_{-n}^{\mu *} e^{-2in(\tau-\sigma)}), \quad (81)$$

where $l^2 = 1/\pi T$. Here the times τ are multiplied only by positive “frequencies” $n > 0$. The conserved momenta corresponding to $Y^\mu, Y^{\mu *}$ are $P_{\alpha\pm}^\mu = \pm T\,\partial_\alpha Y_\pm^{\mu *}$, $\partial^\alpha P_{\alpha\pm}^\mu = 0$, and their commutators have the form:

$$[Y_\pm^\mu(\tau, \sigma'), P_{\tau\pm}^\nu(\tau, \sigma)] = -i\eta^{\mu\nu}\delta(\sigma - \sigma'). \quad (82)$$

The substitution of (81) gives (non-zero) commutators for the modes:

$$[\alpha_m^\mu, \alpha_n^{\nu*}] = [\alpha_{-m}^\mu, \alpha_{-n}^{\nu*}] = -m\delta_{mn}\eta^{\mu\nu}. \tag{83}$$

In TSQ, the Hamiltonian takes the form:

$$H = T^{-1}\int_0^{2\pi} d\sigma\, P_{\alpha\pm} P_\pm^{\alpha*} = H_0 + \sum_{n=1}^{\infty} (\alpha_n^{\mu*}\alpha_{n\mu} + \alpha_{-n}^{\mu*}\alpha_{-n\mu}), \tag{84}$$

where H_0 contains independent on n finite terms. Thus, there is no zero-point energy and the theory is consistent.

The Virasoro algebra operators L_m are automatically normal-ordered $L_0|0\rangle = 0$, and their commutators do not contain anomalies:

$$L_m = L_{m+} + L_{m-} = \sum_{n=-\infty}^{\infty} \alpha_{m+n}^{\mu*}\alpha_{n\mu},$$
$$[L_m, L_n] = (m-n)L_{m+n}. \tag{85}$$

Therefore, the algebra of operators of the Lorentz group is closed and there is no anomaly.

Thus, in TSQ, the bosonic string theory does not contain zero-point energy and has no corresponding anomalies, which means that it does not need a critical measurement D. A similar situation takes place for a fermionic string, where there is also no zero-point energy [8].

Thus, in TSQ, string theory becomes consistent and quite simple and compatible with gravity due to the absence of vacuum energy. But in this form, string theory cannot solve the unification problem, since if there is no zero-point energy of modes, then there will be no conformal anomaly, and therefore there are no reasons for fixing the dimension of space and introducing higher dimensions.

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