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Consistent quantization of systems with positive and negative energy states

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Abstract

The Stueckelberg-Feynman (SF) treatment, where positive energy antiparticles are described as negative energy particles going backward in time, lies on the basis of particle physics, but it was inconsistent with quantum field theory, since led to a negative norm for negative energy states. In the paper a new consistent method of canonical quantization in SF treatment is presented, where norms of all states is positive, since changing the direction of time integration in the action function changes the sign of Lagrangian of antiparticles and momentum. Minimal Lagrangians for complex canonical variables do not lead to the zero-point energy, which partially solves the cosmological constant problem. Causal propagators and amplitudes appear as symmetric chronological products of field operators, which slightly modifies diagram technique. Modified microcausality conditions and proof of spin and statistics theorem are presented, applications to particle physics and condensed media are discussed.

Keywords: *Quantization; zero-point energy; diagram technique; Standard Model; cosmological constant*

Content

1. Introduction

Relativistic quantum theory in the covariant form contains the negative energy particles, which in the Stueckelberg-Feynman (SF) treatment [1,2] (see [3]) evolve only backward in time and describe the positive energy antiparticles evolving only forward in time. However, until now SF treatment was incompatible with quantum field theory (QFT), since in the canonical

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formalism the negative energies led to a negative norm of states. The Dirac's hypothesis about an indefinite metric (see details in [4]), requiring revision of foundations of quantum mechanics, can not be considered as a real solution.

Such an unclear situation led to the standard form of QFT with positive energy particles and antiparticles only (see [5]). This was done by replacing in the field operators the creation (annihilation) operators of the negative energy particles by the annihilation (creation) operators of the positive energy antiparticles: $a^+_{-k} \to b_k$ ($a_{-k} \to b^+_k$). However, in the products of field operators this ansatz led to the diverging zero-point energy, even when it was absent for the negative energy states. Although the normal ordering recipe allowed to hide the problem with zero-point energy in particle physics, the presence of gravitation led to the cosmological constant problem.

One of the ways to solve this problem was to return to the formulation including the negative energy states. A solution of the negative norm problem in the canonical quantization was found by M. Pavšič in 1998 [6]. It was shown that the two-sign Lagrangian and corresponding momenta make the norms of all states positive. But in [6] the physical meaning of the negative energy states remained open.

Recently a consistent method of quantization of systems with two signs of energy states was formulated [7] by complementing the method of Ref. [6] with the treatment of the negative energy states according to SF. In the present paper, this method of quantization is described briefly and then its new applications, including to interacting fields, are considered.

Since the negative energy states evolve backward in time, this property is taken into account from the beginning. In the action function the changing of the time integration direction changes the sign of the Lagrangian and canonical momenta, and then this fact makes positive the norm of states. Thus, SF treatment becomes consistent with QFT.

In Section 2 the consistent form of SF treatment is applied to harmonic oscillators. In Sections 3-5 it is applied to quantum electrodynamics, including interacting fields. In Section 6 various applications of the new method are discussed (gauge fields, gravity, the mass generation, strings, condensed media).

2. Quantization of positive and negative energy states in SF treatment

2.1. SF treatment of negative energy states

In the standard QFT, dealing with the positive energy particles and antiparticles going forward in time only ($t_1 > t_0$), the direction of time integration in the action function should be fixed by the step function:

$$S(t_1,t_0) = \theta(t_1 - t_0)\int_{t_0}^{t_1} L_+ dt + \theta(t_1 - t_0)\int_{t_0}^{t_1} L_{a+} dt\,, \tag{1}$$

where $L_+(q_+, \dot{q}_+)$ and $L_{a+}(q_{a+}, \dot{q}_{a+})$ are the Lagrangians of particles and antiparticles respectively.

In QFT this approach leads to the *non-covariant* diagram technique with a sufficiently larger number of diagrams than in the covariant diagram technique due to the prohibition of integration backward in time for positive energy states. A formal trick by an integral representation for the step function can transform the matrix elements to the same form as in the covariant diagram technique with 4-momentum p_μ, but then there appear the quanta formally having the negative "energy" $-|p_0|$. Thus, the covariant form of the standard QFT inevitable requires to deal with the "negative energy" states, but this can be done consistently only within the framework of SF treatment. How such states can be described in QFT without internal contradictions will be shown below.

A correct transition to SF treatment requires following operations in the second integral of Eq. (1):

$$\int_{t_0}^{t_0+\Delta t} L_{a+} dt = -\int_{t_0+\Delta t}^{t_0} L_{a+} dt = \int_{t_0}^{t_0-\Delta t} (-L_{a+}) dt = \int_{t_0}^{t_0-\Delta t} L_{-} dt; \tag{2}$$

i.e.

(1) taking $t_1 = t_0 + \Delta t$, $\Delta t > 0$, interchange the integration limits by changing the sign of the integral,

(2) using translational symmetry, shift the limits of the integral down to Δt,

(3) introduce the negative energy Lagrangian $L_{-}(q_{-}, \dot{q}_{-}) = -L_{a+}(q_{a+}, \dot{q}_{a+})$.

And finally, let's do the fourth operation:

(4) make similar change in the second step function in (1) and denote $t_1 \to t_0 - \Delta t$:

$$\theta(t_1 - t_0) = \theta[(t_0 + \Delta t) - t_0] = \theta[t_0 - (t_0 - \Delta t)] \to \theta(t_0 - t_1). \tag{3}$$

As a result of steps (1)-(4), the action function $S(t_1, t_0) = S_{+} + S_{-}$ takes the form:

$$S(t_1, t_0) = \theta(t_1 - t_0)\int_{t_0}^{t_1} L_{+} dt + \theta(t_0 - t_1)\int_{t_0}^{t_1} L_{-} dt. \tag{4}$$

Here, the first term with the positive definite Lagrangian L_{+} describes the evolution forward in time ($t_1 > t_0$), and the second term with the negative definite Lagrangian L_{-} describes the evolution backward in time ($t_1 < t_0$), as required by SF treatment.

For relativistic particles the Lagrangians take the form $L_{\pm} = \mp m(1 - \mathbf{v}_{\pm}^2)^{1/2}$. Notice, that the spacetime inversion $x_{+}^{\mu} \to -x_{-}^{\mu}$, $p_{\mu+} \to -p_{\mu-}$, using in SF treatment, changes the sign of 4-momentum, but it does not change the sign of 4-velocity u_{μ}, $p_{\mu\pm} \to \pm m u_{\mu\pm}$. It is important, that the Lagrangian changes sign not because of a dependence on variables, but because of the sign of the time integral in (4). This point distinguishes the present consistent formulation of SF treatment from others.

It is important, that the formulation of particle physics models in SF treatment is directly related to experimental data and does not need to be reformulated in terms of antiparticles. It is enough to know that the initial state of a negative-energy particle with $-p_{\mu}$ represents the final state of an antiparticle with $+p_{\mu}$ and this is the well-known crossing symmetry, one of the basic symmetries of particle physics.

2.2. Harmonic oscillators containing negative energy states

In SF treatment, a system of two harmonic oscillators with coordinates $x_{\pm}$, corresponding to the states of two signs of energy, describes a particle and an antiparticle in the harmonic potential. The canonical formalism gives ($\dot{x}_{\pm} = dx_{\pm}/dt$):

$$\begin{aligned} L_{\pm} &= \pm\frac{m}{2}(\dot{x}_{\pm}^2 - \omega^2 x_{\pm}^2), \quad H_{\pm} = \pm\frac{1}{2m}[p_{\pm}^2 + (m\omega)^2 x_{\pm}^2], \\ p_{\pm} &= \frac{\partial L}{\partial \dot{x}_{\pm}} = \pm m\dot{x}_{\pm}, \quad \dot{p}_{\pm} = -\frac{\partial H_{\pm}}{\partial x_{\pm}} = \mp m\omega^2 x_{\pm}. \end{aligned} \tag{5}$$

The equations of motion $\ddot{x}_\pm + \omega^2 x_\pm = 0$ have the solutions ($w = (2m\omega)^{-1/2}$):

$$x_+ = (2m\omega)^{-1/2}(a_+ e^{-i\omega t} + a_+^* e^{i\omega t}), \qquad x_- = (2m\omega)^{-1/2}(a_- e^{i\omega t} + a_-^* e^{-i\omega t}) \qquad (6)$$

$$p_+ = -i(m\omega/2)^{1/2}(a_+ e^{-i\omega t} - a_+^* e^{i\omega t}), \qquad p_- = -i(m\omega/2)^{1/2}(a_- e^{i\omega t} - a_-^* e^{-i\omega t}) \qquad (7)$$

Quantization gives the commutators:

$$[x_\pm, p_\pm] = i, \quad [a_\pm, a_\pm^*] = 1. \qquad (8)$$

Notice that in the former treatments, due to the positive definite Hamiltonian H_-, the momentum p_- had the form $p_- = m\dot{x}_-$, which then led to the negative defined commutator $[a_-, a_-^*] = -1$ and the negative norm of states: $\langle 1_- | 1_- \rangle = \langle 0 | a_- a_-^* | 0 \rangle = -1$.

In the present treatment, p_- in (5) has the negative sign $p_- = -m\dot{x}_-$ and this provides the positive sign of the commutator $[a_-, a_-^*] = +1$ and the positive norm of states. The ground states, defined as $a_\pm |0\rangle = 0$, lead to the positive norm of the first excited states [6]:

$$\langle 1_\pm | 1_\pm \rangle = \langle 0 | a_\pm a_\pm^* | 0 \rangle = \langle 0 | [a_\pm, a_\pm^*] | 0 \rangle = 1. \qquad (9)$$

The Eq. (8) gives for the ground state wave functions $\psi_{0\pm}$:

$$(m\omega x_\pm + \partial_{x_\pm})\psi_{0\pm} = 0, \quad \psi_{0\pm} = (m\omega/\pi)^{1/4}\exp(-m\omega x_\pm^2/2), \quad \int_{-\infty}^{\infty} \psi_{0\pm}^2 dx_\pm = 1. \qquad (10)$$

Thus, the Hamiltonians (5) in terms of $a_\pm, a_\pm^*$ take the form:

$$H_\pm = \pm\frac{\omega}{2}(a_\pm^* a_\pm + a_\pm a_\pm^*) = \pm\omega(a_\pm^* a_\pm + 1/2). \qquad (11)$$

In terms of the positive energy anti-quanta $H_- \to H_{a+} = \omega(a_{a+}^* a_{a+} + 1/2)$, the total Hamiltonian is $H = \omega(a_+^* a_+ + a_{a+}^* a_{a+} + 1)$, i.e. the zero-point energy (z.p.e.) of the positive energy anti-quanta doubles the total z.p.e.

At the transition to the SF treatment with states with negative energy, the action function remains unchanged and z.p.e. of both modes are summed, doubling the energy of the ground state. This is due to the fact that the integrands can be summed up only after reducing to the same limits of the time integration in the action function and thus, both pictures, with anti-quanta or with negative-energy quanta, give the same result.

Notice that many attempts to cancel z.p.e. of two types of quanta (see [4,6] and many others) required the presence of the negative energy quanta moving forward in time, which are non-physical and prohibited by SF treatment.

Thus, at quantization in the SF treatment of the harmonic oscillator containing states with negative energy, the latter have a positive norm and the theory is consistent.

2.3. Harmonic oscillator with complex coordinates and negative energy states

Let us consider a harmonic oscillator with complex coordinates $q_\pm$, $q_\pm^*$ and containing the negative energy states. In the general case, the Lagrangian includes the symmetrized products of the complex conjugate coordinates and velocities $(\dot{q}_\pm^* \dot{q}_\pm + \dot{q}_\pm \dot{q}_\pm^*)/2$,

$(\dot{q}^*_\pm \dot{q}_\pm + \dot{q}_\pm \dot{q}^*_\pm)/2$. However, we can choose the Lagrangians and the corresponding Hamiltonians also in the minimal form, without such symmetrization:

$$L_\pm = \pm m(\dot{q}^*_\pm \dot{q}_\pm - \omega^2 q^*_\pm q_\pm), \quad H_\pm = \pm\frac{1}{m}(p_\pm p^*_\pm + m^2\omega^2 q^*_\pm q_\pm), \tag{12}$$

where $p_\pm = \partial L_\pm / \partial \dot{q}_\pm = \pm m\,\dot{q}^*_\pm$, $p^*_\pm = \partial L_\pm / \partial \dot{q}^*_\pm = \pm m\,\dot{q}_\pm$. Here the negative signs of p_- and p^*_- make positive the commutator $[a_-, a^*_-]$ and the norm of states. The equations of motion $\dot{p}_\pm = -\partial H_\pm / \partial q_\pm$, $\dot{p}^*_\pm = -\partial H_\pm / \partial q^*_\pm$, or $\ddot{q}_\pm + \omega^2 q_\pm = 0$, $\ddot{q}^*_\pm + \omega^2 q^*_\pm = 0$ give:

$$q = (2m\omega)^{-1/2}(a_+ e^{-i\omega t} + a_- e^{i\omega t}), \quad q^* = (2m\omega)^{-1/2}(a^*_+ e^{i\omega t} + a^*_- e^{-i\omega t}), \tag{13}$$

$$p = i(m\omega/2)^{1/2}(a^*_+ e^{i\omega t} + a^*_- e^{-i\omega t}), \quad p^* = -i(m\omega/2)^{1/2}(a_+ e^{-i\omega t} + a_- e^{i\omega t}). \tag{14}$$

Here $q = q_+ + q_-$, $p = p_+ + p_- = m(\dot{q}^*_+ - \dot{q}^*_-) = im\omega q^*$, and

$$a_\pm = (2m\omega)^{-1/2}(m\omega q_\pm + ip^*_\pm)e^{\pm i\omega t}, \qquad a^*_\pm = (2m\omega)^{-1/2}(m\omega q^*_\pm - ip_\pm)e^{\mp i\omega t}. \tag{15}$$

Quantization gives the commutators:

$$[q, p] = i, \quad [q^*, p^*] = i, \quad [a_\pm, a^*_\pm] = 1. \tag{16}$$

Here the complex conjugated coordinates q and q^* do not commutate: $[q, q^*] = (m\omega)^{-1}$, as well as the momenta $[p^*, p] = m\omega$, but the real-valued products of these complex variables, which are observable, commutate.

The ground states are defined as $a_\pm |0_\pm\rangle = 0$ and:

$$a^*_\pm |n_\pm\rangle = \sqrt{n_\pm + 1}\,|n_\pm + 1\rangle, \quad a_\pm |n_\pm\rangle = \sqrt{n_\pm}\,|n_\pm - 1\rangle, \quad n_\pm = 0, 1, ... \tag{17}$$

The total Hamiltonian H and the number operator N take the form:

$$H = H_+ + H_- = \omega(a^*_+ a_+ - a^*_- a_-), \quad N = a^*_+ a_+ + a^*_- a_-. \tag{18}$$

The lack of z.p.e. in (18) remains unchanged at turning to the anti-quanta also, since $|\langle 0_- | H_- | 0_- \rangle| = \langle 0_{a+} | H_{a+} | 0_{a+} \rangle$.

Thus, the theory of the complex harmonic oscillator containing the negative energy states is consistent and coincides with the SF treatment, the minimal Lagrangians (12) lead to the Hamiltonian (18) without z.p.e.

3. Consistent quantization in the SF treatment of photon and fermion fields

3.1. Photon field

In the gauge $\partial_\mu A^\mu_\pm = 0$ the Lagrangians for the photon field $A^\mu_\pm$ of positive and negative energy states ($k^0 = \pm\omega_{\mathbf{k}}$) take the form ($F^{\mu\nu}_\pm = \partial^\mu A^\nu_\pm - \partial^\nu A^\mu_\pm$):

$$L_\pm = \mp\frac{1}{4}\int d^3x\, F^\pm_{\mu\nu} F^{\mu\nu}_\pm = \pm\frac{1}{2}\int d^3x\, \partial_\mu A^\nu_\pm\, \partial^\mu A_{\nu\pm}. \tag{19}$$

At $\mathbf{k}=(0,0,k^3)$ and $A_3=A_0=0$, the transverse physical components form a complex field $A=(A_1+iA_2)/\sqrt{2}$, $A^*=(A_1-iA_2)/\sqrt{2}$. The Lagrangian (19) in minimal form and the Hamiltonian take the form similar to the complex scalar field:

$$L_\pm=\pm\int d^3x\,\partial_\mu A_\pm^*\partial^\mu A_\pm,\quad H_\pm=\pm\int d^3x(\pi_\pm\pi_\pm^*+\nabla A_\pm^*\nabla A_\pm). \tag{20}$$

Here $\pi_\pm(x)=\pm\partial_t A_\pm^*$, $\pi_\pm^*(x)=\pm\partial_t A_\pm$ and A describes the circularly polarized photons. It diagonalizes the Hamiltonian and the helicity:

$$\Lambda_\pm^0=\pm i\int d^3x\,(A_\pm^*\pi_\pm^*-\pi_\pm A_\pm). \tag{21}$$

The latter is in fact a chiral charge and photons with $\Lambda^0=\pm1$ are like a particle and a charge-conjugate antiparticle.

The field equations $\partial_\mu\partial^\mu A=0$, $\partial_\mu\partial^\mu A^*=0$ give:

$$A=\sum_k(a_k e^{-ikx}+a_{-k}e^{ikx}),\;\; \pi=i\sum_k\omega_k(a_k^* e^{ikx}+a_{-k}^* e^{-ikx}), \tag{22}$$

where $\sum_k=\int d^3k[(2\pi)^3 2\omega_k]^{-1/2}$, $a_{\pm k}^*, a_{\pm k}$ are the creation-annihilation operators for $\Lambda_+^0=+1$. For the commutators and observables this gives:

$$[A(\mathbf{x},t),\pi(\mathbf{x}',t)]=i\delta^3(\mathbf{x}-\mathbf{x}'),\quad [a_{\pm k},a_{\pm k'}^*]=\delta^3(\mathbf{k}-\mathbf{k}'), \tag{23}$$

$$H=\int d^3k\,\omega_k(a_k^*a_k-a_{-k}^*a_{-k}),\quad \Lambda^0=\int d^3k(a_k^*a_k+a_{-k}^*a_{-k}). \tag{24}$$

The ground states, defined as $a_{\pm k}\left|0\right\rangle=0$, do not contain z.p.e. and helicity. Due to the invariance of the vacuum, the vanishing of its energy in one of frames and in one of gauges is true for all frames and gauges.

The photon field, having two transverse physical components $(0,A_1,A_2,0)$ in the given frame, can have in other frames all four components $A_\mu=e_\mu^1A_1+e_\mu^2A_2$ and the polarization vector ε_μ, where the tetrads (e_μ^1,e_μ^2) are the constant kinematic factors.

3.2. Fermion field

The SF treatment is natural for the spin ½ fermions, where the Dirac field ψ includes both the positive energy spinor $\psi_+^\alpha\sim u_p^\alpha$ and the negative energy spinor $\psi_-^\alpha\sim v_p^\alpha$. The Lagrangian L_- in (4) not need in additional minus sign, since this sign is effectively taken into account in the normalization conditions for the spinors v_p^α :

$$L_\pm=\int d^3x\left(\frac{i}{2}[\bar\psi_\pm\gamma^\mu(\partial_\mu\psi_\pm)-(\partial_\mu\bar\psi_\pm)\gamma^\mu\psi_\pm]-m\bar\psi_\pm\psi_\pm\right). \tag{25}$$

The corresponding Hamiltonian $H=H_++H_-$, charge operator $Q=Q_++Q_-$, and the Dirac equation:

$$H_{\pm}=\frac{i}{2}\int d^{3}x\,[\psi_{\pm}^{+}(\partial_{t}\psi_{\pm})-(\partial_{t}\psi_{\pm}^{+})\psi_{\pm}],\qquad Q_{\pm}=\int d^{3}x\,\psi_{\pm}^{+}\psi_{\pm},$$
$$\gamma^{\mu}i\partial_{\mu}\psi-m\psi=0 \tag{26}$$

lead to the modes (with $E_{p}=(\mathbf{p}^{2}+m^{2})^{1/2}$ and $u_{p}^{\alpha+}u_{p}^{\alpha'}=v_{p}^{\alpha+}v_{p}^{\alpha'}=\delta^{\alpha\alpha'}E_{p}/m$):

$$\psi(x)=\psi_{+}+\psi_{-}=\sqrt{2m}\sum_{\alpha p}(b_{p\alpha}u_{p}^{\alpha}e^{-ipx}+b_{-p\alpha}v_{p}^{\alpha}e^{ipx}),$$
$$\psi^{+}(x)=\psi_{+}^{+}+\psi_{-}^{+}=\sqrt{2m}\sum_{\alpha p}(b_{p\alpha}^{+}u_{p}^{\alpha+}e^{ipx}+b_{-p\alpha}^{+}v_{p}^{\alpha+}e^{-ipx}). \tag{27}$$

The wave functions of fermions must be anti-symmetric, and hence the field operators must be anti-commuting. At rearranging the field operators in commutators their product changes the sign and the commutator turn to the anti-commutators. Thus, the fermion field is quantized by the equal time anticommutators ($\pi=i\psi^{+}$):

$$\left\{\psi^{\alpha}(\mathbf{x},t),\pi^{+\alpha'}(\mathbf{x}',t)\right\}=i\left\{\psi^{\alpha}(\mathbf{x},t),\psi^{+\alpha'}(\mathbf{x}',t)\right\}=i\delta^{3}(\mathbf{x}-\mathbf{x}')\delta^{\alpha\alpha'},$$
$$\left\{b_{\pm p\alpha},b_{\pm p'\alpha'}^{+}\right\}=\delta^{3}(\mathbf{k}-\mathbf{k}')\delta_{\alpha\alpha'}. \tag{28}$$

The Eqs. (26)-(27) give for observables:

$$H=\sum_{\alpha}\int d^{3}p\,(b_{p\alpha}^{+}b_{p\alpha}-b_{-p\alpha}^{+}b_{-p\alpha})E_{p},\qquad Q=\sum_{\alpha}\int d^{3}p\,(b_{p\alpha}^{+}b_{p\alpha}+b_{-p\alpha}^{+}b_{-p\alpha}). \tag{29}$$

Thus, the fermionic ground states, defined as $b_{\pm p\alpha}\left|0\right\rangle=0$, do not contain z.p.e. and zero-point charge.

In the standard treatment of QFT there appeared z.p.e. in the Dirac theory, and moreover, with a negative sign. However, in fact, this was the result of an inaccurate transition to the picture of antiparticles.

Действительно, первая неточность была допущена при замене в (13) на , после чего античастицы с положительной энергией, которые должны описываться , продолжали описываться , спинором с отрицательной энергией

Вторая неточность была допущена, когда не было учтено, что начальное состояние частицы с отрицательной энергией является конечным состоянием античастицы с положительной энергией.

At first, in (27) the term $b_{-p\alpha}v_{p}^{\alpha}$ was replaced by $d_{p\alpha}^{+}v_{p}^{\alpha}$ and the positive energy antiparticles, which should be described by the spinor u_{p}^{α}, continued to be described by v_{p}^{α}, the negative energy spinor. Second, it should be taken into account, that the initial state of a negative energy particle is the final state of a positive energy antiparticle:

$$\left\langle n_{-}-1\right|b_{-p\alpha}\left|n_{-}\right\rangle=\left\langle n_{a+}\right|d_{p\alpha}^{+}\left|n_{a+}-1\right\rangle,$$
$$\left\langle n_{-}\right|b_{-p\alpha}^{+}\left|n_{-}-1\right\rangle=\left\langle n_{a+}-1\right|d_{p\alpha}\left|n_{a+}\right\rangle. \tag{30}$$

4. Commutators and propagators, microcausality and spin-statistics theorem

4.1. Commutators of fields and a modified microcausality condition

A complex scalar field $\phi = \phi_+ + \phi_-$ has the modes similar to (22) and its commutators do not vanish at spacelike intervals:

$$[\phi_\pm(x'), \phi^*_\pm(x)] = \sum_k e^{\mp ik(x'-x)} = D_\mp(x'-x). \tag{31}$$

In fact, the only microcausality condition, following from the quantum-mechanical measurability, is that the commutator of the complex scalar field $\phi_\pm$ and its momentum $\pi_\pm = \pm\partial_t \phi^*_\pm$ must vanish at spacelike intervals and must coincide by the equal time commutator (23). Direct calculation gives an expression through the Pauli-Jordan function $D(x'-x)$ obeying these requirements:

$$[\phi(x'), \pi(x)] = i\partial_t D(x'-x). \tag{32}$$

For a fermion field the momentum is $\pi = i\psi^+$ and an analogue of (32) is the standard anticommutator vanishing at spacelike intervals:

$$\{\psi(x')\bar{\psi}(x)\} = (i\gamma^\mu \partial_\mu + m)iD(x'-x). \tag{33}$$

It coincides with the equal time anticommutators (28).

Thus, at consistent quantization in the SF treatment, the microcausality conditions become modified: it disappears behind the light cone not the commutator of the complex conjugated field operators (31), but the commutator of canonically conjugated operators (32). This is in full agreement with the requirements of quantum mechanics for observables, since in the case of complex fields the observables are described by the bilinear products of the field operators only. From the physical point of view this means that a “coordinate” and corresponding “momentum” can not be measured simultaneously only when they describe the causally related events with a timelike or lightlike interval between them.

4.2. The causal propagators of fields

The standard chronological product of operators T_+ arranges the positive energy operators in order of increasing time from right to left:

$$T_+[A_+(t')B_+(t)] = \begin{cases} A_+(t')B_+(t), & t' > t, \\ B_+(t)A_+(t'), & t' < t. \end{cases} \tag{34}$$

In the SF treatment, it also appears the operator T_-, inverse to T_+, i.e. defined as ordering the negative energy operators in reverse order, as time decreases from right to left:

$$T_-[A_-(t')B_-(t)] = \begin{cases} B_-(t)A_-(t'), & t' > t, \\ A_-(t')B_-(t), & t' < t. \end{cases} \tag{35}$$

The symmetric chronological ordering operator $\hat{T}$ can be defined as their product $\hat{T} = T_+T_-$, with properties $\hat{T}A_+ = T_+A_+$, $\hat{T}A_- = T_-A_-$. It acts selectively on two signs of energy operators, ordering them in the inverse order and at $t' > t$ gives:

$$\hat{T}[A_+(t')B_+(t)A_-(t')B_-(t)] = [A_+(t')B_+(t)][B_-(t)A_-(t')]. \tag{36}$$

In the SF treatment, the SF causal propagators appear naturally as the symmetric chronological products ($\hat{T}$ -products):

$$\begin{aligned} &iD_c(x'-x) = \langle 0|\hat{T}[\phi(x')\phi^*(x)]|0\rangle = \\ &= \langle 0|\phi_+(x')\phi_+^*(x)\theta(t'-t) + \phi_-(x')\phi_-^*(x)\theta(t-t')|0\rangle = \\ &= \sum_k \left[\theta(t'-t)e^{-i\omega_k(t'-t)} + \theta(t-t')e^{i\omega_k(t'-t)}\right]e^{i\mathbf{k}(\mathbf{x}'-\mathbf{x})}. \end{aligned} \tag{37}$$

The SF causal propagator for fermions is:

$$\begin{aligned} &iS_c(x'-x) = \langle 0|\hat{T}[\psi(x')\bar{\psi}(x)]|0\rangle = \\ &= \langle 0|\psi_+(x')\bar{\psi}_+(x)\theta(t'-t) + \psi_-(x')\bar{\psi}_-(x)\theta(t-t')|0\rangle = \\ &= -(i\gamma\partial + m)D_c(x'-x). \end{aligned} \tag{38}$$

Thus, in the SF treatment the time-symmetric $\hat{T}$ - product replaces the former artificial rules for constructing of the causal propagators.

4.3. New aspects of the spin and statistics theorem

In the standard QFT the proof of the spin and statistics theorem was based on two arguments. The first was the positive energy condition for a spinor field, leading to the anticommutators, and the second was the microcausality condition requiring disappearance of the (anti)commutators of field operators at spacelike intervals.

In Section 3.2, the spin ½ field was quantized by using anticommutators, assuming that it is a fermion field. But here the anticommutators did not exclude the negative energies and the introduction of anticommutators was not directly related with the sign of energy. As it was shown in Section 4.1, in the SF treatment the microcausality conditions become weaker, since the boson field commutators do not vanish at spacelike intervals, like the causal propagators.

In this regard, there appear new aspects of the proof of the spin and statistics theorem. The proof should be based on more general arguments, since the specific properties of field operators give only a sufficiency condition, but not a necessity condition**.**

5. Interacting fields and the modified diagram technique

At the SF quantization, two types of contributions to the free Hamiltonian are additive $H_0 = H_{0+} + H_{0-}$ and the time dependence of a free field φ is $\varphi_\pm(t) = e^{\mp iH_{0\pm}t}\varphi_\pm(0)e^{\pm iH_{0\pm}t}$. The time evolution of the interacting fields is described in terms of the interaction Hamiltonian H_I. At the additive contribution of the positive and negative energy interaction terms $H_I = H_{I+} + H_{I-}$ the time-evolution is given by the evolution operator:

$$U(t,t_0) = \hat{T}\exp\left(-i\int_{t_0}^{t} dt'[\theta(t-t_0)H_{I+}(t') + \theta(t_0-t)H_{I-}(t')]\right). \tag{39}$$

where $\hat{T} = T_+T_-$ is the time-symmetric operator defined in (36).

In the general case there is a mixed part of the interaction vertex $\tilde{H}_{I+}\tilde{H}_{I-}$, leading particularly to the pair creation and annihilation, and $H_I = H_{I+} + H_{I-} + \tilde{H}_{I+}\tilde{H}_{I-}$. For this reason (39) can be written as a general $\hat{T}$ - product:

$$U(t,t_0) = \hat{T}\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right). \tag{40}$$

In the perturbation theory the S -operator, defined as $\hat{S} = U(\infty,-\infty)$, takes the form:

$$\hat{S} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\int_{-\infty}^{\infty} dt_1 ... \int_{-\infty}^{\infty} dt_n \hat{T}[H_I(t_1)...H_I(t_n)]. \tag{41}$$

The modified Wick theorem for two-point function is:

$$\hat{T}[\phi(x')\phi^*(x)] =: \phi(x')\phi^*(x) : + \langle 0|\hat{T}[\phi(x')\phi^*(x)]|0\rangle, \tag{42}$$

where $:A:$ means normal ordering. For example, in the electron-positron loop diagram the $\hat{T}$ - product $\hat{T}[H_I(x')H_I(x)]$ gives the standard formulas:

$$\begin{aligned}\hat{T}[H_I(x')H_I(x)] &= T_+[\psi_{j+}(x')\bar{\psi}_{m+}(x)]\gamma_{\mu mn}T_-[\psi_{n-}(x)\bar{\psi}_{i-}(x')]\gamma^{\mu}_{ij} = \\ &= -S_{c+}(x'-x)_{jm}\gamma_{\mu mn}S_{c-}(x-x')_{ni}\gamma^{\mu}_{ij}, \qquad t' > t,\end{aligned} \tag{43}$$

$$\begin{aligned}\hat{T}[H_I(x')H_I(x)] &= T_-[\psi_{j-}(x')\bar{\psi}_{m-}(x)]\gamma_{\mu mn}T_+[\psi_{n+}(x)\bar{\psi}_{i+}(x')]\gamma^{\mu}_{ij} = \\ &= -S_{c-}(x'-x)_{jm}\gamma_{\mu mn}S_{c+}(x-x')_{ni}\gamma^{\mu}_{ij}, \qquad t' < t.\end{aligned} \tag{44}$$

For the interacting fields there appear three dissimilarities of the new form of the SF treatment with respect to the standard diagram technique:

(1) there are negative energy modes instead of antiparticles and the time integration for them goes to the past,
(2) the chronological ordering depends on the sign of energy and is time-symmetrical,
(3) the operators in the vertices do not (anti)commutate $\phi(x)\phi^*(x) \neq \phi^*(x)\phi(x)$.

In fact, these dissimilarities were effectively taken into account in the standard diagram technique by means of some formal rules (normal ordering, constructing of propagators, etc.). This explains why the previous results mainly coincide with the results of the new method.

6. The Standard Model fields and some other applications

6.1. The massless gauge fields and gravitons

The massless non-Abelian vector gauge field A^a_μ (spin 1) and the graviton field (spin 2) have two transverse physical states. There is an axial symmetry, and at circular polarization the free Hamiltonian and the helicity are diagonal. For this reason, in the weak field approximation their quantization is similar to the photon field case. Further taking into account the nonlinearity and internal symmetries of these fields does not change the conclusion about the lack of z.p.e. Really, their nonlinear contributions are proportional to the coupling constants and should decrease at the weak field limit, while z.p.e., the energy of fluctuations of free fields $\omega_k/2$, does not depend on the interaction constants.

In the case of gravitons, the effective coupling constant $\sim \Lambda / \Lambda_g$ is weak up to the Planch energy Λ_g. At the Planck length l_g, where the gravitational radius of particles is close to their wavelength, there is a strong gravitational redshift of the proper frequencies for external observers. A rapid decreasing of the proper frequencies at distances of order l_g due to the gravitational freezing of the proper times for external observers, leads to the decreased contribution of the nonlinear effects to the S-matrix.

The conclusion that at quantization of the massless non-Abelian fields and the graviton field in one a frame, in the transverse gauge and the weak-coupling limit z.p.e. does not arise, remains valid for all frames and gauges due to the invariance of the vacuum.

6.2. New aspects of the mass generation mechanism

In the Standard Model, the mass generation mechanism was based on the spontaneous symmetry breaking in the system of gauge fields and a doublet of two complex scalar fields, when the latter reduced to one real scalar field.

In the SF treatment the frequency decomposition of an initial complex scalar field has the form $\chi \sim \chi_+ f + \chi_- f^*$ and, at quantization, the vacuum do not contain z.p.e. However, the standard replacement of the complex field by the real scalar field with the frequency decomposition $\chi \sim \chi_+ f + \chi_+^* f^*$ leads to z.p.e. This means that this simple replacement is not painless and has serious consequences making the theory inconsistent.

Thus, the standard mechanism of mass generation should be improved so that the neutral scalar field contains the states of both particle and antiparticle, even if they are practically indistinguishable.

There is another serious problem with the scalar field - power-law growth of its loop corrections. These corrections become very large at the Planck energy and, therefore, the theory with a fundamental scalar field will remain inconsistent even after the inclusion of the gravity effects. For this reason the scalar field can in reality be an effective field with a composite scalar boson [7]. The various modifications of the standard mass generation mechanism will be discussed elsewhere.

6.3. A consistent and simple string theory

The theories of relativistic strings are based on the fact that at quantization they lead to a critical dimension of spacetime: $D = 26$ for bosonic and $D = 10$ for fermionic strings (including superstrings). In fact the critical dimension is a consequence of the presence of the diverging z.p.e. of string modes (see details in [7]). It follows from the requirement to cancel anomalies arising from the residual contribution of z.p.e. of string modes after their "regularization".

Let's consider SF quantization of strings. The action for a boson string is:

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu = -T\int d^2\sigma \partial_\alpha Y_\mu^* \partial^\alpha Y^\mu, \tag{45}$$

where $Y^\mu = (X_R^\mu + iX_L^\mu)/\sqrt{2}$, $Y^{\mu *} = (X_R^\mu - iX_L^\mu)/\sqrt{2}$. In the SF treatment, firstly, instead of a superposition of modes $\tau \pm \sigma$, moving right or left, each with positive and negative energies, one of the modes is selected as the basic one, for example, the right one with positive energy $Y_+^\mu(\tau - \sigma)$, moving forward in time only. Secondly, the left positive mode $Y_+^\mu(\tau + \sigma)$ is described as the right mode with negative energy $Y_-^\mu(\tau - \sigma)$ moving backward in time. Thirdly, the action function should be rewritten by taking into account these two points as:

$$S=-T\int_{\tau_0}^{\tau_1}d\tau\int d\sigma[\theta(\tau_1-\tau_0)\partial_\alpha Y_+^*\partial^\alpha Y_+-\theta(\tau_0-\tau_1)\partial_\alpha Y_-^*\partial^\alpha Y_-] \quad (46)$$

For closed strings the frequency modes for the complex coordinates take the form:

$$\begin{aligned}Y^\mu(\tau-\sigma)&=Y_0^\mu+\frac{il}{2}\sum_{n=1}^\infty\frac{1}{n}(\alpha_n^\mu e^{-2in(\tau-\sigma)}+\alpha_{-n}^\mu e^{2in(\tau-\sigma)}),\\ Y^{\mu*}(\tau-\sigma)&=Y_0^{\mu*}-\frac{il}{2}\sum_{n=1}^\infty\frac{1}{n}(\alpha_n^{\mu*} e^{2in(\tau-\sigma)}+\alpha_{-n}^{\mu*} e^{-2in(\tau-\sigma)}),\end{aligned} \quad (47)$$

where $l^2=1/\pi T$. Here the times τ are multiplied only to the positive "frequencies" with $n>0$. Conserved currents of $Y^\mu, Y^{\mu*}$ are $P_{\alpha\pm}^\mu=\pm T\,\partial_\alpha Y_\pm^{\mu*}$, $\partial^\alpha P_{\alpha\pm}^\mu=0$, their commutators have the form:

$$[Y_\pm^\mu(\tau,\sigma'),P_{\tau\pm}^\nu(\tau,\sigma)]=-i\eta^{\mu\nu}\delta(\sigma-\sigma'). \quad (48)$$

Substitution of (47) gives (nonzero) commutators for the modes:

$$[\alpha_m^\mu,\alpha_n^{\nu*}]=[\alpha_{-m}^\mu,\alpha_{-n}^{\nu*}]=-m\,\delta_{mn}\eta^{\mu\nu}. \quad (49)$$

The Hamiltonian takes the form:

$$H=T^{-1}\int_0^{2\pi}d\sigma\,P_{\alpha\pm}P_\pm^{\alpha*}=H_0+\sum_{n=1}^\infty(\alpha_n^{\mu*}\alpha_{n\mu}-\alpha_{-n}^{\mu*}\alpha_{-n\mu}), \quad (50)$$

where H_0 contains independent on n finite terms. Thus, there is no z.p.e. and the theory is consistent.

The operators L_m of the Virasoro algebra are automatically normal ordered $L_0|0\rangle=0$ and their commutators do not contain an anomaly:

$$L_m=L_{m+}+L_{m-}=\sum_{n=-\infty}^{\infty}\alpha_{m+n}^{\mu*}\alpha_{n\mu},\ [L_m,L_n]=(m-n)L_{m+n}. \quad (51)$$

Therefore, the algebra of operators of the Lorentz group is closed also without anomalous terms.

Thus, the theory of the boson string does not contain z.p.e. and is free of corresponding anomalies, which means that it does not need in a critical dimension D. A similar situation is for the fermionic string, which is also free of z.p.e. Therefore, in the SF treatment string theory is consistent and sufficiently simpler, but it cannot solve the problems of unification.

6.4. SF treatment of quasiparticles and anti-quasiparticles in condensed media

Methods of quantization of systems of harmonic oscillators and, in a broader form, methods of quantum field theory naturally applied in the theory of condensed media. The fact that instead of particles and antiparticles there appear quasiparticles and anti-quasiparticles (or holes), changes the specifics only, but the main advantages of field theory methods remain the same and appear very useful. In the case of the new consistent form of the SF treatment, the situation is the same and in the future it may turn out to be useful for solving corresponding problems of condensed media.

In systems where there is z.p.e. in the ground state, there may not be new features, but this method will become one of the convenient ways of description in some cases. However, in systems where the conditions for the absence of z.p.e. are satisfied, this fact will either be easier to describe, or, if it was not previously known, it will be predicted and discovered.

Particularly interesting can be applications in the systems with nontrivial properties and effects, such as graphene and other two-dimensional structures. In them, states without z.p.e. are already known, as well as states described by the Dirac equation, so here a new method, natural just for such cases, can become a useful working tool.

7. Conclusion

The SF quantization of relativistic systems with the negative energy states, including the relativistic fields and strings, is consistent, since the norm of all states is positive at taking into account that the Lagrangian changes the sign when the time integration in the action function changes direction. In this treatment, QFT with particles of both signs of energy, evolving in opposite time directions, is equivalent to the theory in terms of positive energy antiparticles, but is free of z.p.e. The absence of the contribution of free fields to the vacuum energy partially solves the cosmological constant problem.

For fermionic fields, the new method of quantization turns out to be natural and the standard canonical formalism changes insignificantly. The canonical formalism for complex bosonic fields undergoes a more significant change, since their canonically conjugate field operators become non-commutative. But, since the observables of free fields are described by their real valued bilinear products, there are no problems from the physical point of view. In the standard treatment such a situation was with fermionic field operators.

For interacting fields, SF treatment introduces the time-symmetric $\hat{T}$-products of operators, which directly leads to SF causal propagators and modifies the diagram technique by preserving the final results of the standard QFT. The method revises the microcausality conditions, mass generation mechanism and the proof of the spin and statistic theorem, makes consistent and simpler the string theories. It also may be useful in the theory of condensed media, especially for two dimensional systems.

Thus, the new method of SF quantization, consecutively providing SF treatment, can be considered as a step towards the finite and consistent QFT.

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