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Time-symmetric relativistic and quantum theories.
1. Time-symmetric theories of special and general relativity

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Abstract

In relativistic theory, particles have negative energy pairs, which were considered unphysical, but Dirac (1930) linked them with antiparticles, Zisman (1940) and Stueckelberg (1941) showed that they formally evolve backward in ordinary time t, and Feynman (1949) developed a covariant diagram technique based on them. However, the description of antiparticles in terms of negative energy states remained incomplete, and a number of problems arose. In this article, a time-symmetric relativity theory (TSRT) is formulated based on this interpretation, eliminating the previous problems. In TSRT, the proper time of a negative-energy particle is measured in its rest frame, the basis of which has negative energy and also evolves backward along t axis. Specifically, in antihydrogen, a negative-energy proton realizes the basis of such a frame. In TSRT, the principle of relativity is extended to reference frames going backwards along t-axis, and in time-symmetric special relativity, the symmetry group becomes the general Lorentz group O(1,3), which includes 4-inversion. Including translations leads to the general Poincaré group. In time-symmetric general relativity, the O(1,3) group acts locally, and the Einstein equations and their solutions remain unchanged. In TSRT, the 4-vectors of the probability current and interaction currents change sign under 4-inversion, and in time-symmetric relativistic quantum theory, this resolves a number of problems (see Articles 2 and 3).

Key words: negative energy particles, Zisman-Stueckelberg-Feynman interpretation, antiparticles, time inversion, 4-inversion, general Lorentz group, special relativity, general relativity

Content

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1. Introduction

One of the most important achievements of relativistic quantum theory was the prediction of antiparticles in an attempt to interpret negative energy states inevitable appearing in the covariant form of the theory.

In Dirac's hypothesis for fermions, called the hole theory [1], it was assumed that the negative energy levels were filled, and the holes in this background appeared as antiparticles. But in the hole theory the particles, filling negative energy states, were considered also as moving *forward* in ordinary time t, which in fact contradicts the relativity theory, where they can go only *backwards* in this time.

The discovery of the latter property in 1940–1941 by Zisman [2] and Stueckelberg [3] was a key step towards an adequate description of negative energy particles. This property allowed Feynman to create in 1948–49 the propagator approach and to develop the covariant diagram technique [4], which underlie particle physics (see [5,6]).

The Zisman-Stueckelberg-Feynman (ZSF) interpretation is based on a property of the relativity theory's formalism where, at considering trajectories globally in event space, positive-energy antiparticles evolving forward in ordinary time are equivalent to negative-energy particles evolving backward in that time. This possibility is not a hypothesis and manifests itself in particle physics through crossing symmetry. However, this interpretation was applied incompletely, leading to problems in the theory, particularly the negative probability problem in boson theory. A deeper reason was that the previous form of relativity theory, developed for positive-energy states, was not adapted to negative-energy states.

In particular, in the special relativity (SR) only ordinary inertial frames of reference (IFR) were introduced, going forward in time, due to which the symmetry group was restricted to the orthochronous Lorentz group $O^{\uparrow}(1,3)$, excluding reflection of the time axis. As a result, the theory that introduced (formally) negative-energy particles did not provide rest frames for them in which their proper times and other proper characteristics could be measured.

To consistently follow ZSF interpretation, it was therefore necessary to introduce (also formally) rest frames for the negative energy particles, which must also go backwards along t axis with their time axis t_{-} opposite to t.

The physical realization of such reference frames is demonstrated by the description of the motion of a positron in an antihydrogen atom, where the antiproton at the center is the basis of a reference frame with a spherical coordinate system. At describing the motion of positron in terms of negative-energy electrons, the antiproton is also described as a negative-energy proton and realizes the basis of a reference frame moving backward t axis.

This extension of relativity theory by including reference frames going to the lower light cone, with a corresponding extension of the principle of relativity, underlies *the time-symmetric relativity theory* (TSRT) or *time-symmetric relativity* (TSR), the formulation of which is the aim of the present article. TSR includes time-symmetric special relativity (TSSR) in flat spacetime and time-symmetric general relativity (TSGR) in the general case.

In TSSR the transformation group becomes $O(1,3)$, known as the general Lorentz group [7]. It includes the rotation group and the discrete transformations of coordinates, leaving the form $x_0^2 - x_1^2 - x_2^2 - x_3^2$ invariant. Incorporating translations of the origin of coordinates leads to the general Poincaré group, leaving the form $dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2$ invariant.

In TSSR, global and local inertial frames are introduced in flat spacetime, while in TSGR they are introduced in curved spacetime with the local group $O(1,3)$. Einstein's equations and their solutions remain unchanged. The 4-inversion does not change the sign of the velocity, but it does change the sign of the acceleration, and the convergence of world lines in the direct system appears as their separation in the inverse system.

In Article 2, it is shown that the time reflection is described by a unitary operator T, which is equivalent to a combination of Wigner time reversal, described by an anti-unitary operator T_W, and charge conjugation C. Thus, in TSR 4-inversion PT replaces the CPT_W transformation of the former relativistic quantum theory. As it is well known, T_W reverses processes by permuting the initial and final states, but does not change the time axis or the sign of energy, whereas in TSR, T reverses the signs of energy and currents. Therefore, here T_W is an active transformation when some vectors are reflected at the same time axis, whereas T is a passive transformation when the vectors are the same, but the time axis is reflected.

In TSR, fourth component of the probability current has two signs for particles evolving in two directions of the time axis, which is important in relativistic quantum theory. It eliminates the need to replace the probability current with interaction currents, which led to absurdity for truly neutral particles, where a zero interaction current was treated as a zero probability current. An absurd situation was also in Dirac's theory, where a particle current in the backward time direction corresponded a positive probability current of fermions. In TSR, the 4-vector of the probability current is the same for all free relativistic particles, has the same sign as the energy-momentum vector, and changes sign upon 4-inversion.

In Section 2 ZSF interpretation is described and a more consistent approach to taking into account its requirements, leading to TSR, is considered. In Section 3 foundations of TSSR, and in Section 4 foundations of TSGR are described. In the articles 2 and 3 foundations of time-symmetric relativistic quantum theory, mechanics and field theory, are formulated. The initial ideas and preliminary materials on the time-symmetric approach were presented in [8]. A more detailed presentation of TSR and its applications are given in the book [10].

2. From ZSF interpretation to the time-symmetric special relativity

2.1. On the motion of negative energy particles backward in ordinary time

In relativity theory, a set of events at different times and different points in space is defined in a four-dimensional event space, and in flat spacetime, events are fixed in one of the ordinary IFR K_+ with a time axis t. For this purpose, a global system of physical coordinates is constructed with the origin at the event O using mutually resting standard scales and clocks. The coordinate axes are constructed based on four unit vectors $\mathbf{e}_\mu$, $\mu = 0,...,3$, which usually form an orthogonal frame. Contravariant components dx^μ of the expansion coefficients of the 4-vectors of translation $d\mathbf{x}$ along the frame vectors $d\mathbf{x} = dx^\mu \mathbf{e}_\mu$ give the coordinates of this vector. The scalar products of the unit vectors give the metric tensor $g_{\mu\nu} = \mathbf{e}_\mu \cdot \mathbf{e}_\nu$, and the covariant components dx_μ are equal to the scalar products $d\mathbf{x}$ and unit vectors $\mathbf{e}_\mu$:

$$\mathbf{e}_\mu \cdot d\mathbf{x} = \mathbf{e}_\mu \cdot \mathbf{e}_\nu dx^\nu = g_{\mu\nu} dx^\nu = dx_\mu . \tag{1}$$

Proper time interval $d\tau_\pm$ between two close events on the trajectory of a particle, measured in its rest frame K'_+, is expressed through the time interval dt between the same events in K_+, where this particle moves with velocity $v^i = dx^i / dt$ (here $i = 1,2,3$ and $v^2 = v_i v^i$). This dependence follows from thought experiments taking into account the invariance of the speed of light and the space-time interval ds, and in a static coordinate system, where the local bases are mutually resting, is given by the expressions:

$$ds^2 = c^2 d\tau_\pm^2 = d\mathbf{x} \cdot d\mathbf{x} = dx_0 dx^0 + dx_i dx^i , \tag{2}$$

$$d\tau_{\pm} = \pm dt\sqrt{1-v^2/c^2}, \quad \frac{dt}{d\tau_{\pm}} = \pm\frac{1}{\sqrt{1-v^2/c^2}} = u^0_{\pm}. \tag{3}$$

In the standard relativity theory, the negative sign in (3) was discarded as unphysical, assuming that for real particles and antiparticles, their proper time τ always increases with the time of the reference frame t. This is true, and in nature, there are no objects that correspond to the negative sign in (3). In TSR, which will be formulated below, the situation is the same, and real objects also correspond to the positive sign in (3).

However, after the theory of relativity was created, the situation in physics changed, and pairs for all particles, their antiparticles, were discovered. Particles and antiparticles were found to be related by a new symmetry – charge conjugation symmetry. Since the number of real states doubled, it became possible to avoid discarding the previous negative sign "extra" states in (3), but to somehow use them to describe antiparticles. This possibility, realized in ZSF interpretation in a restricted form, is realized in TSR in a complete form, and for this reason below the main relationships of the theory will be presented for both signs in (3).

The main consequence of the relationships (3) is that the 4-velocity components:

$$u^{\mu}_{\pm} = \frac{dx^{\mu}}{ds_{\pm}} = \frac{dx^{\mu}}{cd\tau_{\pm}} \tag{4}$$

have two signs not only along the spatial axes, but along the time axis also.

Proper time of a particle $\tau_{\pm}$ is measured by the clock that *comoves* it, and therefore $\tau_{\pm}$ always increases along the particle's trajectory. However, the evolving direction of $\tau_{\pm}$ and the evolving direction of t coincide only for τ_{+} on the trajectories going forward in time t. A nontrivial property of the relativity theory, discovered by Zisman (1940) [2] and Stückelberg (1941) [3], is the fact that there are trajectories with τ_{-} in (3), formally going backwards in time t, when τ_{-} increases and t decreases along the trajectory due to a negative sign in (3).

This property manifested itself only in relativistic physics, where the concept of event's time is richer than in nonrelativistic physics and is divided into two parts. First, there is the invariant proper time of the particle $\tau_{\pm}$, which is measured in its rest frame. Second, there are the times t, t', t'',… of this event in different inertial frames, where the particle moves at different velocities. These times are relative, since the same interval $\Delta\tau_{+}$, according to (3), corresponds to different intervals Δt, $\Delta t'$, $\Delta t''$.

The difference *in the magnitudes* of time intervals Δt measured in different reference frames was the beginning of the creation of special relativity, and this opened a new era in physics. The possibility of also different *signs* of these time intervals was first studied in 1940–41 [2,3], but neither then nor since has this led to the formulation of the problem of a corresponding extension of relativity theory. Such an extension, including a new class of reference frames going into the lower light cone, correspondingly extending the group of transformations and formulating TSR, will be described below.

The difference in signs of Δt and $\Delta\tau_{-}$ alone would have no physical meaning, remaining a formal possibility with no relation to reality. The purely mathematical aspects of such an extension have been studied for a long time [7]. However, it acquired physical meaning after the discovery of two additional nontrivial facts. The first, key one, will be consider here, while the second one will be discussed in the next section.

The first key fact is that with the growth of proper time along the trajectory, the intervals $\Delta\tau_{\pm} > 0$ correspond not only to two signs of the intervals of ordinary time $\pm\Delta t$, but also to two signs of the particle's energy, i.e. *with a change in sign* Δt *the sign of the energy also changes.* This follows from the definition of 4-momentum:

$$p^{\mu}_{\pm} = mcu^{\mu}_{\pm} = mc\frac{dx^{\mu}}{ds_{\pm}} = m\frac{dx^{\mu}}{d\tau_{\pm}}, \tag{5}$$

which together with (3) gives the expression for energy $E_{p\pm} = cp^0_{\pm} = mc^2u^0_{\pm}$:

$$E_{p\pm} = mc^2\frac{dt}{d\tau_{\pm}} = \pm\frac{mc^2}{\sqrt{1-v^2/c^2}}. \tag{6}$$

This clearly shows that the difference in the signs of the energies of the particles is determined only by the factor $dt/d\tau_{\pm}$ and the negative sign in (3) leads to a negative sign of the energy in (6). Thus, the form $E_{p\pm}t_{\pm}$ is invariant under reflection of the time axis.

At changing the time direction of the trajectory only, the spatial components of the 4-velocity $u^i_{\pm}$ (4) do not change, since before and after turning the trajectory back in time, the particle continues to go in the positive direction of x^1-axis, $dx^1/d\tau_{\pm} > 0$, since with growth $\tau_{\pm}$, the coordinate x^1 continues to grow.

Thus, in relativity theory, the positive energy particles go only forward, and the negative energy particles go only backward in ordinary time t.

This, in particular, demonstrates the physical inconsistency of numerous hypotheses that introduce negative energy states evolving forward in time. The first and most famous of these was Dirac's hole theory [1]. The fact that this and similar hypotheses contradict relativity theory and are therefore unacceptable has not yet been sufficiently recognized and, therefore, has not been reflected in scientific and educational literature.

2.2. ZSF interpretation completed by rest frames of negative energy particles leads to TSR

Let's consider the second of the nontrivial facts related to the negative energies. If the formalism of relativity theory includes particles with such energies that also evolve backward in ordinary time, as discussed in the previous section, then the question arises: what is the physical meaning of such states?

The answer turned out to be unexpected and nontrivial: in nature, there are only positive energy particles and antiparticles going only forward in ordinary time t, but *in the covariant form of relativity theory, antiparticles appear as the negative energy particles, going backward in time* t. This unusual property of the theory's formalism is the second key fact that gave physical meaning to backward evolution. It underlies ZSF interpretation and, later TSR in the more systematic form, since negative energy states turned out to be the simplest and most practical way to describe real antiparticles.

The formulation of relativistic quantum theory on the basis of ZSF interpretation, where there are no antiparticles, but only particles of two energy signs, moving forward or backward in time depending on this sign, was given by Feynman in 1948-1949 [4]. At the same time, he developed on this basis the propagator method and the covariant diagram technique, as well as the method of path integrals, which have become effective and practical tools of physics.

The transition to ZSF interpretation simplifies the description of systems of particles and antiparticles, since instead of two types of objects with opposite charges, there is only one type - particles, and antiparticles are described as some state of the particles, differing only kinematically, in the signs of energy and the evolving direction of their proper times.

This simplification of the formalism is analogous to the transition to highly compact and efficient representations in two fundamentally important formalisms. In one case, this involves introducing a single complex number instead of two real numbers - even though complex numbers don't exist in reality, they are simpler and more efficient to operate with than two real numbers. In the other case, this involves the transition to event space, where time is a fourth (imaginary) dimension in addition to the three dimensions of space. Here, real space is three-dimensional, but the transition to four-space makes the theory simpler and more efficient, allowing, in particular, its generalization to curved spacetime.

Thus, the ZSF interpretation is not a hypothesis, but a consequence of relativistic kinematics, allowing for the effective use the basic properties of the relativity theory's formalism to describe antiparticles. In quantum theory, it allows for the theory to be formulated in a manifestly covariant form, which makes it similar to what Minkowski did.

ZSF interpretation lies at the heart of relativistic quantum mechanics, but even within this science, it has been applied extremely inconsistently to date. This has led to problems, including internal contradictions in the theory. The well-known of these is the negative probability problem in theory of bosons.

The main contradiction in the standard formulation of relativistic quantum mechanics based on the ZSF interpretation was that, while the theory operated with the proper time of negative energy particles, but did not provide a rest frame for these particles where it would be measured. Another problem was the incompatibility of ZSF interpretation with standard quantum field theory, where only positive-energy states exist, and the vacuum is the lowest energy state. Fermion antiparticles, however, are still described by spinors for negative-energy states due to their extreme convenience, despite the internal contradiction in such approach.

Therefore, in order to consistently provide ZSF interpretation, it is necessary to take into account the two circumstances mentioned above:

a) the negative energy particles have rest frames that also go backwards in ordinary time, and the trajectories of their elements cross the hypersurface of simultaneity $t = const$ from top to bottom along the time axis;

b) therefore, the relativity theory must be extended to those reference frames that go into the lower light cone, and whose coordinates are related to the ordinary coordinates through time inversion, and in the general case through 4-inversion.

The question of correctly accounting for these circumstances was raised in the author's earlier publications, leading to the conclusion that a time-symmetric extension of relativity theory is necessary. The article will then outline the basics of TST.

An example of the physical realization of a reference frame moving backward along t axis was given in the Introduction. This is the case the antihydrogen atom described in the ZSF interpretation. Here, the basis of the reference frame with spherical coordinates is the antiproton, described as a negative energy proton, and a reference frame with such a basis certainly evolves backward along t axis. The antiproton is accompanied by a positron, described as a negative energy electron, also going backward along t axis.

Invariance condition $x_0^2 - x_1^2 - x_2^2 - x_3^2$ defines a group of coordinate transformations that include rotations and reflections of axes. This group is the general Lorentz group $O(1,3)$, which includes the 4-inversion $x_-^\mu = -x^\mu$. Therefore, in TSSR, the group of coordinate transformations becomes $O(1,3)$.

At including translations, which consist of choosing another event as the origin of the coordinate system and the rotation point of the corresponding axes, there appears the general Poincaré group, leaving invariant the form $dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2$. The consequences of this group will be considered in [10]. The mathematical aspects of the group $O(1,3)$ are discussed in [7] and some facts about it necessary for physical applications are presented also in [10].

Let us note that TSR, which extends ZSF interpretation to reference frames comoving the negative-energy particles, also contains no new hypotheses. Its mathematical formalism is a natural generalization of the ordinary formalism of relativity theory for forward time evolution to backward time evolution. Moreover, it was already contained in the formalism of ordinary relativity theory, but was considered unphysical and discarded. TSR gives this part a physical meaning by associating it with antiparticles, resulting in a theory that describes systems of positive-energy particles and antiparticles in a covariant and compact form by more effectively exploiting the capabilities of the existing formalism.

3. Time-symmetric special relativity based on general Lorentz group

3.1. Transition from the full to the general Lorentz group

The standard formulation of ZSF interpretation was based on the restriction to ordinary reference frames K_+, whose elements consist of positive-energy particles moving only forward in time t. Coordinate transformations between inertial frames were therefore restricted to the orthochronous (full) Lorentz group $O^{\uparrow}(1,3)$, acting to the trajectories directed to the upper light cone and allowing for reflections of spatial coordinates by preserving the time axis.

Only symmetry with respect to the replacement of particles with antiparticles and invariance with respect to the charge conjugation operation *C* were introduced, sometimes together with the parity transformation *P*. Time reversal T_W was introduced as an active operation, when vectors change and processes reverse in time with the same time axis, i.e., the time interval changes sign when the sequence of events changes. Therefore, the reversal of time intervals led to the reversal of velocities and other time derivatives in reversible processes with permuted initial and final states.

At using the ZSF interpretation, the restriction to ordinary reference frames, as noted above, leads to an internal contradiction in the theory, since positive energy particles had rest frames in which their proper time can be measured, while the rest frames for negative energy particles were not allowed.

If some of particles can formally evolve backward in time along their worldlines, then all physical aspects of kinematics and dynamics, also formally, must be adapted to such a radical change in temporal evolution. Local elements of an ordinary reference frame, going forward in time, in particular, a clock measuring the proper time of ordinary particles, cannot be comoving to a particle going backward in time. Clocks measuring the proper time of a negative-energy particle must be *comoving* it and therefore must also evolve backward in ordinary time. This means that they must consist of negative-energy particles and be an element of a reference frame evolving backward in time. Physically, this means that we are talking about reference frames in which all elements consist of antiparticles described in terms of negative energies.

In the usual consideration of symmetry regarding the replacement of particles with antiparticles, a hidden asymmetry was allowed, since the reference frame remained the same, i.e., consisting of ordinary particles. Complete symmetry requires replacing this frame with a reference frame consisting of antiparticles, since we are talking about the replacement of the world with an antiworld. This method of description is obviously inapplicable in a situation where the basis of the reference frame consists of a mixture of particles and antiparticles.

Thus, the transition to ZSF interpretation requires extending the group of coordinate transformations of reference frames up to $O(1,3)$ with $t_- = -t$ in the particular case and 4-inversion $x_-^{\mu} = -x^{\mu}$ in the general case. In this group, in addition to the two proper transformations of frames with mutually inverse time axes, there are also transitions between them via time axis inversion.

The ordinary proper Lorentz transformations with the standard matrix Λ_{ν}^{μ} from the orthochronous group $O^{\uparrow}(1,3)$ relate the coordinates of an event in two reference frames K_+ and K'_+, moving forward in ordinary time:

$$x_+^\mu{}' = \Lambda^\mu_{\nu+} x_+^\nu. \tag{7}$$

In two rest frames of the negative energy particles K_- and K'_-, going backward in ordinary time, the coordinates of the event are also related by proper Lorentz transformations with a matrix $\Lambda^\mu_{\nu-}$ from $O^\uparrow(1,3)$, formally the same as $\Lambda^\mu_{\nu+}$:

$$x_-^\mu{}' = \Lambda^\mu_{\nu-} x_-^\nu. \tag{8}$$

Coordinates of an event x_-^μ related to ordinary coordinates x_+^μ of the same event in K_+ through improper transformations $\Lambda^\mu_{\nu-+}$, $\Lambda^\mu_{\nu+-}$ from the antichronic group $O^\downarrow(1,3)$, including the inversion of the time axis:

$$x_+^\mu{}' = \Lambda^\mu_{\nu+-} x_-^\nu, \qquad x_-^\mu{}' = \Lambda^\mu_{\nu-+} x_+^\nu. \tag{9}$$

With the same origins and parallel axes of both coordinate systems, i.e. with zero relative 3-velocity of the two reference frames, the 4-inversion of the global coordinate system in the event space has the form:

$$x_-^\mu = \Lambda^\mu_{\nu-+} x_+^\nu = -x_+^\mu, \quad \Lambda^\mu_{\nu-+} = -\delta^\mu_\nu. \tag{10}$$

In this case, the time axes of these coordinate systems are directed in opposite directions:

$$x_-^0 = -x_+^0. \tag{11}$$

Invariance under the group $O(1,3)$ follows from a generalization of the principle of relativity: the laws of physics are identical for particles of two energy signs in reference frames with corresponding energy signs moving in space and time in mutually opposite directions. The equation of motion for a positive-energy particle in a reference frame moving forward in time is the same as the equation of motion for a negative-energy particle in a reference frame moving backward in time with coordinates associated with the ordinary 4-inversion (10).

At a 4-inversion (10), which relates the coordinates of two classes of reference frames, the signs of all polar 4-vectors change. Therefore, if, remaining in K_+, we turn to a particle going backward in time, the sign of the 4-momentum changes $p_-^\mu \to -p_+^\mu$, as the sign of the 4-vector of the probability current $j_-^\mu \to -j_+^\mu$.

If we begin by considering states where particles of both energy signs are at rest in space with zero 3-velocities $v_-^i = -v_+^i = 0$, then even in this case they *are not at rest in time.* Their world lines, being parallel, run in opposite directions of the time axis, since the 4-velocity of the first particle is $u_+^\mu = (1,0)$, and that of the second $u_-^\mu = (-1,0)$.

It should be noted that the transition to TSR fundamentally changes ZSF interpretation, since the latter is generalized and becomes applicable to reference frames of both classes, each of which describes the motion of particles with two energy signs. The sign of (2) now depends on the combination of the directions of the particle's time evolution and the reference frame in which the motion is described. Particularly, for particles going backward in ordinary time t_+ this sign is negative in K_+, but becomes positive in K_-, since $dt_- = -dt_+$:

$$d\tau_- = -dt_+\sqrt{1-\mathbf{v}^2/c^2}, \quad d\tau_- = dt_-\sqrt{1-\mathbf{v}^2/c^2}. \tag{12}$$

3.2. Densities in orientable hypersurfaces of simultaneity

Under 4-inversion, polar 4-vectors change sign, and this property should hold for any objects with negative energy. However, this wasn't ensured in relativistic quantum theory, which led to some of the problems. For this reason, let's consider the requirements of TSR for the properties of most important vectors and the tensors formed from them.

The first one is the normal vector to the hypersurface of simultaneity $t = const$, which in terms of physical time is directed toward the future for K_+ and back to it in K_-. K_+ This vector has a single nonzero component $\mathbf{e}_0$, directed along or against the axis x^0, and its direction determines the sign of the oriented 3-volume element.

The invariant (proper) energy-momentum density $\varepsilon \geq 0$, measured in the rest frame of the matter (or a small section of the field) in a given volume element, is positive-definite for objects of both energy signs and is defined as:

$$\varepsilon = c\frac{dp^0_{(0)}}{dV_{(0)}}. \tag{13}$$

Here $dV_{(0)}$ is the proper 3-volume element, $p^0_{(0)}$ is the proper energy. The 4-vector of the energy-momentum density $\boldsymbol{\varepsilon}$ is then defined as the product of ε and the 4-velocity of the particle $\mathbf{u} = \mathbf{dx}/d\tau$, where $\mathbf{dx}$ is the 4-vector of the shift:

$$\boldsymbol{\varepsilon} = \varepsilon\mathbf{u} = \frac{dp^0_{(0)}}{dV_{(0)}}\frac{\mathbf{dx}}{d\tau}. \tag{14}$$

Components of the energy-momentum tensor $T^{\mu\nu}$ for an ideal fluid, including also the (proper) pressure p, have the form:

$$T^{\mu\nu} = [(\varepsilon + p)u^\mu u^\nu - g^{\mu\nu}p]. \tag{15}$$

In the absence of pressure, the components $T^{\mu\nu}$ are quadratic in the 4-velocity and do not change under the 4-inversion. In particular, the component $T^{00} = \varepsilon u^0 u^0 \geq 0$ is positive-definite for matter and field of both energy signs, since $\varepsilon \geq 0$. This fact is important for the gravity theory in TSGR, where Einstein's equations remain unchanged.

In TSR, the sign of the 4-component of the vectors of various currents, including the probability current, changes as the direction of current along the time axis changes. Particle trajectories intersect the hypersurface $x^0 = const$ from the bottom to up at positive energies and from the top to down at negative energies, and the sign of the current indicates the direction of intersection along the time axis.

Probability w is a positive-definite scalar quantity and does not change under 4-inversion, remaining positive-definite even at negative energies. In K_+ 4-vector $\mathbf{j}$, the probability current density, is product of its proper probability density $\rho_{(0)} = dw/dV_{(0)} \geq 0$ and particle's 4-velocity of drift $\mathbf{u}$:

$$\mathbf{j} = \frac{dw}{dV_{(0)}}\frac{\mathbf{dx}}{d\tau} = \rho_{(0)}c\mathbf{u} = \frac{1}{m}\rho_{(0)}\mathbf{p}. \tag{16}$$

Thus, the coordinates of the 4-vector $\mathbf{j}$ are transformed similarly as the coordinates of the 4-translation vector $\mathbf{dx}$ and, like electromagnetic current, are proportional to coordinates of the 4-momentum $\mathbf{p}$. The 4-volume is invariant $d^4x = cd\tau dV_{(0)} = cdtdV$ and therefore for positive energies (16) reduces to the standard expression:

$$\mathbf{j} = \frac{dw}{dV}\frac{\mathbf{dx}}{dt} = \rho\frac{\mathbf{dx}}{dt}. \tag{17}$$

The coordinates $\mathbf{j}$ are defined as $j_{\pm}^{\mu} = \mathbf{j}_{\pm} \cdot \mathbf{e}^{\mu}$ and, in particular, the component $j_{\pm}^{0}$ is proportional to the energy of particles in the current and also has two signs:

$$j_{\pm}^{0} = \rho_{(0)} \frac{dx_{\pm}^{0}}{d\tau} = \rho_{(0)} c u_{\pm}^{0} = \rho_{(0)} \frac{p_{\pm}^{0}}{m}. \tag{18}$$

Thus, K_{+} the sign $j_{\pm}^{0}$ coincides with the sign of the energy of the particles, while $j_{+}^{0} > 0$ describes the probability current density of the positive energy particles moving along t_{+} axis, and $j_{-}^{0} < 0$ describes the probability current density of the negative energy particles moving backward to t_{+}.

The coordinates of the 4-vector of electromagnetic current density j_{q}^{μ} are determined similarly j^{μ} and differ only in the replacement of the probability density ρ by the charge density $\rho_{q} = dq / dV$, the sign of which also depends on the sign of the charge $q = \pm |q|$:

$$j_{q}^{\mu} = \rho_{q} \frac{dx^{\mu}}{dt} = \frac{dq}{dV_{(0)}} \frac{dx^{\mu}}{d\tau} = \rho_{q(0)} c u^{\mu} = \rho_{q(0)} \frac{p^{\mu}}{m}. \tag{19}$$

The sign j_{q}^{μ} here depends on the combination of signs of p^{μ} and charge.

In the particular case of a positively charged particle with $\rho_{q(0)} > 0$, which evolves in both time directions, the time component of the current density j_{q}^{0} is analogous to the probability current density (18). It also has two signs, depending on the direction along the axis t_{+} in which the charge current intersects the hypersurface $x^{0} = const$, i.e., it depends on the sign of the energy:

$$j_{q\pm}^{0} = \rho_{q(0)} c u_{\pm}^{0} = \rho_{q(0)} \frac{p_{\pm}^{0}}{m}. \tag{20}$$

In general, when both signs of charge are considered, the same value of current density (20) corresponds to two different situations. Positive current density $j_{q}^{0} > 0$ describes both the forward motion of a positive charge and the backward motion of a negative charge. A negative current density $j_{q}^{0} < 0$, on the other hand, describes both the backward motion of a positive charge and the forward motion of a negative charge.

In ZSF interpretation, these properties of the current density are used: if $j_{q}^{0} > 0$ corresponds to an ordinary particle with a positive charge, then the same particle, going backward in time, corresponds to $j_{q}^{0} < 0$, which can also be interpreted as an antiparticle with a negative charge going forward in time.

3.3. Time-symmetric relativistic mechanics of free particles

In the ordinary SR, the action functions along the trajectories of free particles and antiparticles of positive energy are the same and in K_{+} have the form:

$$S = -mc^{2} \int_{\tau_0}^{\tau_1} d\tau_{+} = -mc^{2} \int_{t(\tau_0)}^{t(\tau_1)} dt \cdot \frac{d\tau_{+}}{dt} = \int_{t_0}^{t_1} dt\, L_{+}, \tag{21}$$

$$L_+ = -mc^2 \frac{d\tau_+}{dt} = -mc^2\sqrt{1-v^2/c^2}, \tag{22}$$

where $t_1 = t(\tau_1)$, $t_0 = t(\tau_0)$, with $\tau_1 > \tau_0$ and $t_1 > t_0$. During interactions, the particle and antiparticle differ in charge and their states are related through charge conjugation.

At describing an antiparticle as a negative-energy particle, its charge remains the same as that of the particle, and only the energy signs and evolutionary directions change. The time derivative with t is negative: $d\tau_- / dt = -\sqrt{1-v^2/c^2}$ and in K_+ growth of the proper time of negative-energy particles along the trajectory occurs with decreasing t, i.e., $\tau_1 > \tau_0$ leads to $t_1 < t_0$. In K_- action function has the same form as in (21):

$$S = -mc^2 \int_{\tau_0}^{\tau_1} d\tau_- = -mc^2 \int_{t_{0-}}^{t_{1-}} dt_- \cdot \frac{d\tau_-}{dt_-} = \int_{t_{0-}}^{t_{1-}} dt_- L_{(-)}, \tag{23}$$

$$L_{(-)} = -mc^2 \frac{d\tau_-}{dt_-} = -mc^2\sqrt{1-v_-^2/c^2}. \tag{24}$$

We can write this action function in K_+ by two ways. In the simplest way, using (3), we can write the integral directly in terms of ordinary time t:

$$S = -mc^2 \int_{t(\tau_0)}^{t(\tau_1)} dt \frac{d\tau_-}{dt} = \int_{t_0}^{t_1} dt\, L_-, \qquad L_- = mc^2\sqrt{1-v^2/c^2}, \quad t_1 < t_0. \tag{25}$$

Notice that here the differences from (21) are in the negative sign of the Lagrangian $L_- = -L_+$ and in the backward integration in ordinary time with $t_1 < t_0$. For a more detailed describing such a transition, it is necessary to perform in (23) an inversion $t_- = -t$ and, due to translational invariance, shift the limits of the integral by $t_0 + t_1$, which then yields:

$$S = \int_{-t_0}^{-t_1} d(-t) L_{(-)} = \int_{t_1}^{t_0} dt\, L_-, \;\; t_1 > t_0. \tag{26}$$

Renaming the limits of integration so that the initial point is t_0, and the final point is t_1, we obtain (25) with $t_1 < t_0$. Thus, at 4-inversion, the Lagrange function changes sign, which is natural, since its sign determines the sign of the energy:

$$E_\pm = p_{i\pm} v^i - L_\pm, \quad p_{i\pm} = \frac{\partial L_\pm}{\partial v^i}, \;\; v^i = \frac{dx^i}{dt}, \tag{27}$$

taking into account that at 4-inversion the sign of the velocity v^i does not change ($i = 1,2,3$).

The action function for a particle with states of two energy signs can thus be written in the conditional form:

$$S = \theta(t_1 - t_0)\int_{t_0}^{t_1} dt\, L_+ + \theta(t_0 - t_1)\int_{t_0}^{t_1} dt\, L_-. \tag{28}$$

For, $t_1 \neq t_0$ this notation is exact, but for, $t_1 \to t_0$ clarification is necessary, since the θ -function experiences a discontinuity, and its derivative yields $\delta(t_1 - t_0)$. Since, for $t_1 \to t_0$ definite integrals in (28) vanishes, this notation becomes correct we suppose that at the limit $t_1 \to t_0$ the integral vanishes. This situation is similar to giving physical meaning to the square of the δ -function when one of the δ -functions is left finite. In our case, the δ -function is left finite, and the integral vanishes. Below we will assume that the time integrals are written in this time-symmetric form.

In the general case of a system with two energy signs, the generalized coordinates $q_\pm = (q_\pm^1,...,q_\pm^n)$ and their generalized velocities $\dot{q}_\pm$ define the Lagrangian function $L_\pm = \pm L(q_\pm, \dot{q}_\pm)$, where $L(q_+, \dot{q}_+)$ for a particle of positive energy, a $L(q_-, \dot{q}_-)$ is the same Lagrangian function, but with the variables q_+ and $\dot{q}_+$ replaced by q_- and $\dot{q}_-$. The action then has the form (28). In the canonical formulation, the generalized momenta $p_\pm$, as well as the Hamiltonian functions $H_+ = H(q_+, \dot{q}_+)$ and $H_- = -H(q_-, \dot{q}_-)$ are defined as:

$$p_\pm = \frac{\partial L_\pm}{\partial \dot{q}_\pm}, \quad H_\pm = p_\pm \dot{q}_\pm - L_\pm, \quad H_\pm = \pm H(q_\pm, p_\pm). \tag{29}$$

The Hamilton-Jacobi equations following from (28)-(29) are of the form:

$$\pm\frac{\partial S}{\partial t} + H_\pm = 0, \quad \frac{\partial S}{\partial t} + H(q_\pm, p_\pm) = 0. \tag{30}$$

For extended objects and fields, the densities of the Lagrange function L and the Hamiltonian function are introduced H:

$$\begin{aligned} S &= \int d^4x[\mathsf{L}_+(q_+, \dot{q}_+) + \mathsf{L}_-(q_-, \dot{q}_-)] = \\ &= \int d^4x[p_+\dot{q}_+ + p_-\dot{q}_- - \mathsf{H}_+(q_+, p_+) - \mathsf{H}_-(q_-, p_-)]. \end{aligned} \tag{31}$$

Here, the signs of $\mathsf{L}_\pm$ and $\mathsf{H}_\pm$ are determined by the directions of integration - forward or backward in time. The corresponding equations of motion are:

$$\dot{q}_\pm = \frac{\partial \mathsf{H}_\pm}{\partial p_\pm}, \quad \dot{p}_\pm = -\frac{\partial \mathsf{H}_\pm}{\partial q_\pm}. \tag{32}$$

Time-symmetric formulations of mechanics and field theory also describe systems with particles and antiparticles in the nonrelativistic approximation. The usual restriction to particles was due to the rarity of low-energy antiparticles and the exclusion of practically unused parts from the formalism of nonrelativistic theory. In quantum theory, the TSSR leads to time-symmetric relativistic quantum mechanics and quantum field theory based on the general Lorentz group, which are described in articles 2 and 3.

3.4. Time-symmetric relativistic mechanics with interactions

Let's consider the simplest case of a particle with an electric charge. In K_+ the particle and antiparticle have opposite charges and attract each other, their world lines converge, and at colliding, the pair annihilates. When a pair is created, they still attract each other, moving away with a large opposite momentum. In K_- these two processes are reversed.

Therefore, the inclusion of interactions leads to an asymmetry with respect to the reflection of the time axis for a particular interaction process, but there is a higher-level symmetry, *crossing symmetry*. This implies the presence of another process, where particle trajectories are transferred from one light cone to another, with a change in the signs of their four-momenta.

At the presence of interactions mixing two types of states of different energy signs, their canonical variables can be combined:

$$q = q_+ + q_-, \quad p = p_+ + p_- \tag{33}$$

and the general action function can be written in a compact form:

$$S = \int_{t_0}^{t_1} dt\, L(q,\dot{q}) = \int_{t_0}^{t_1} dt\, [p\dot{q} - H(q,p)]. \tag{34}$$

Here, the time integral limits are chosen depending on the energy sign for terms with identical variables, while the ordinary limits are used for mixed interaction terms. The time-symmetric Hamilton-Jacobi equation has the form:

$$\frac{\partial S}{\partial t} + H(q,p) = 0. \tag{35}$$

For extended objects and fields, we have:

$$S = \int d^4x\, \mathsf{L}(q,\dot{q}) = \int d^4x\, [p\dot{q} - \mathsf{H}(q,p)]. \tag{36}$$

A matrix method of combining generalized coordinates of two types is also possible:

$$q = \begin{pmatrix} q_+ \\ q_- \end{pmatrix}, \quad p = \begin{pmatrix} p_+ \\ p_- \end{pmatrix}, \tag{37}$$

which may be convenient for certain systems. This method of description naturally leads to the isospin formalism at describing particle-antiparticle pairs and is often used in particle physics.

4. Time-symmetric general relativity

4.1. Physics in local inertial and inhomogeneous extended frames of reference

In TSSR, spacetime is flat and events are described in global inertial frames of reference (GIFRs). GIFRs consist of local bases, aligned at a given moment and resting, and associated with them tetrads. Local systems of physical coordinates are formed from standard scales, joined in a certain order, and synchronized standard clocks, and the symmetry group of these coordinates, both local and global, is $O(1,3)$.

For separate accelerated objects, local inertial frames of reference (LIFRs) are introduced that instantaneously comove them, where the group transformations $O(1,3)$ become local. *The principle of locality*, according to which, during accelerated motion of objects, all local relativistic effects are reduced to velocity effects, allows one to correctly describe all phenomena involving accelerated objects within the framework of special relativity while remaining within a GIFR with world time. This gives rise to specific properties of the set of LIFRs along the trajectories of accelerated objects during single, multiple, or continuous transitions of accelerated objects from one LIFR to another. In particular, the non-inertial dilation of their proper times relative to the world time of GIFR *is absolute*, as is known from the clock paradox.

Of fundamental importance from a physical point of view is the fact that the bases, scales, and clocks of all LIFRs forming an inhomogeneous extended frame of reference (IEFR) must coexist simultaneously in at least one a GIFR at the time $t = const$ of this GIFR. Under this condition, the spatial axes formed by the set of LIFR are projected onto the hypersurface $t = const$, and their temporal axes are projected onto the axis t. As a result, an effective metric appears in this set of LIFR, leading to inhomogeneous relativistic acceleration effects, which is a step toward the transition to general relativity even in flat spacetime.

A classic, clear example of this is the specificity of describing phenomena in a rotating disk, noted by Einstein early in the development of general relativity. Here, there is a physically distinct GIFR - the rest frame of the center - since in this GIFR, the disk is axially symmetric, and the description is extremely simplified (the transition to GIFR moving along the axis preserves this symmetry but only complicates description without changing physical results). At each instant in the GIFR time $t = const$, the set of LIFR instantaneously comoving the particles of the disk forms a IEFR. From the point of view of the observers in GIFR, the effects of relativistic kinematics in this IEFR, the contraction of scales along the circumferences, and the dilation of proper times, are as stronger as farther from the center this LIFR.

From the perspective of the formalism of general relativity, the fact that such an effective metric arises from relativistic effects in the non-uniform relativistic frame is irrelevant. In general relativity, what matters is the very existence of a non-trivial metric leading to non-uniform relativistic effects, independent of the physical causes that give rise to them. These could be the mutual motions (including accelerations) of local elements of the reference frame against the background of flat spacetime (non-uniform inertial frames of reference and non-inertial frames of reference), the curvature of spacetime itself against the background of stationary elements of the reference frame, or both physical causes combined.

At changing the structure of spacetime due to a source of gravity, the equivalence principle allows one to describe phenomena in reference frames at rest in a gravitational field similarly to the description of phenomena in accelerated reference frames in the absence of gravity. In this case, there is also a distinguished extended reference frame in which the center of inertia of the gravitational field source is at rest. Therefore, here too, LIFRs are introduced, not moving ones, but instantaneously at rest relative to the field source, in which the local transformation group is also $O(1,3)$. Then, from these mutually resting LIFRs, an IEFR is constructed, also at rest relative to the field source, and in this IEFR, not an effective metric, but a real metric corresponding to the gravitational field appears.

If we also introduce non-uniformity in the reference frame due to the relative motion of the LIFRs included in the IEFR, then the metric will contain contributions from both the gravitational field and kinematic effects due to the relative motion of the LIFRs.

The TSGR expands this picture of relativistic effects by introducing corresponding changes related to the presence of two types of objects and two similar types of reference frame bases that evolve in different directions along the time axis, globally and locally. These changes will be described in more detail in the book [10].

4.2. Field equations and gravitational effects in time-symmetric general relativity

The components of the metric $g_{\mu\nu}$ and the energy-momentum tensor $T^{\mu\nu}$ do not change under 4-inversion, as do the Ricci tensor and scalar, in which differentials of coordinates enter quadratically. This means that Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}, \tag{38}$$

where $\kappa = 8\pi G / c^3$, also do not change under the 4-inversion. Thus, in TSGR the gravitational field equations and their solutions, as well as the effects of gravity, remain essentially the same as in ordinary general relativity.

The picture of the physical effects of gravity, however, changes due to differences in the kinematics in the two types of reference frames. Gravity in the static case manifests itself as the mutual approach of initially rested ordinary particles. At considering this process in the "inverse" reference frame, the beginning and end of the process are reversed, resulting in the mutual removal of these particles, with opposite momenta at the initial instant, and will continue to move away until they come to rest at the points where they would immediately come to rest. Thus, the same process in the same static metric is described in two reference frames with mutually inverse initial and final states.

The worldlines of antiparticles in gravitational fields are the same as those of ordinary particles, and therefore, at considering negative-energy particles, they remain the same; only the direction of particle evolution along the same worldlines changes, due to the rearrangement of the initial and final events. In particular, a static metric is, by definition, time independent, and changing the time direction only alters the direction of sequence of events in the worldlines.

For gravitons, helicity plays the role of a chiral charge, and gravitons with opposite helicities, like photons, act as a particle and antiparticle, to which the ZSF interpretation should then be applied [9].

The comments given in this part of the article are preliminary and will be further adjusted, since the extension of GR with the transition to a local group $O(1,3)$ requires a more detailed consideration, which will be given in subsequent publications, in particular, in the book [10].

5. Conclusion

In relativity theory, negative-energy particles, presenting in the covariant formulation, evolve only backward in time. This indicates that the hole theory, which assumes such particles to evolve forward in time, is incompatible with relativity theory and is therefore inconsistent.

A consistent interpretation of such states is the ZSF interpretation, in which negative-energy particles evolve backward in ordinary time and describe positive-energy antiparticles evolving forward in time. This interpretation has previously led to a number of controversies due to its incorrect application, primarily due to its restriction to ordinary inertial frames K_+ with an orthochronous Lorentz group $O^{\uparrow}(1,3)$, which does not include time axis inversion.

At a correct applying the ZSF interpretation, it is necessary to include rest frames of negative-energy particles, where their proper times are measured. Such reference frames consist of antiparticles, which, in terms of negative-energy particles, also evolve backward in time. The group of transformations of physical coordinates then becomes the general Lorentz group $O(1,3)$ including the 4-inversion. Including translations of the origin leads to the general Poincaré group. This expansion of the set of admissible reference frames leads to TSR, in which both particles and their rest frames can evolve in both directions of time, but only in accordance with the sign of the energy. TSR removes the internal contradictions in quantum theory associated with previous particular applications of the ZSF interpretation.

In TSSR, global and local IFRs are introduced in flat space-time with global and local transformation groups $O(1,3)$, and in TSGR also in curved space-time, where this group, acting in LIFRs, becomes local.

In TSGR, Einstein's equations remain unchanged, since the metric tensor, the stress-energy density tensor, and the Ricci tensor do not change sign under the 4-inversion. However, acceleration changes sign under the 4-inversion, and for the same static metric, the kinematic effects of gravity are asymmetric with respect to the reflection of the time axis. This kinematic effect is due to the different directions of tracing events along the same worldlines.

In papers 2 and 3 [9] the extensions according to TSR of relativistic quantum mechanics and quantum field theory, respectively, will be presented. A more detailed presentation of TSR and its applications to quantum theory is given in the book [10].

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