QUANTUM AND GRAVITATIONAL PHYSICS
5:027, 13 p., 02.02.2024; doi:10.9751/QGPH.5-027.8433
ISSN 2181-0486; EISSN 2181-0508

Foundations of time-symmetric physics. 2. Quantum field theory

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Abstract

Time symmetric theory (TST), described in the first article and based on the equivalence principle of particle physics, is applied to quantum fields and time-symmetric quantum field theory is formulated. This principle is based on the Zisman-Stückelberg-Feynman interpretation, where antiparticles are described as negative energy particles moving backward in time, and generalizes it to the rest frames of such particles also moving backward in time. In TST, relativistic fields are described by complex field operators, and observables are automatically normally ordered. For this reason, at time-symmetric quantization (TSQ) of fields, the vacuum energy and charge disappear and there is no cosmological constant problem. In TST, the probabilities of states are positive, and the probability currents can have two signs depending on the direction of evolution along the time axis. The operator of interaction current is defined as the product of the probability current and the matrix of interaction constants. Applications of TST to the Standard Model fields and graviton field are considered. TSQ correctly describes the observable effects, since in terms of ordinary time it leads to a standard diagram technique, transforming mathematical recipes of the latter into logical consequences of physical principles.

Keywords: quantization, zero-point energy, diagram technique, Standard Model, cosmological constant, gauge fields, electroweak theory

Content

1. Introduction

Time-symmetric theory (TST), based on the principle of equivalence of particle physics, was applied to relativistic quantum mechanics in the first paper [1]. In the present paper, TST is applied to quantum field theory (QFT) and *time-symmetric QFT* is formulated, the foundations of which were considered in preliminary form in the papers [2].

This equivalence principle is a further generalization of the Zisman-Stückelberg-Feynman (ZSF) interpretation widely used in particle physics [3,4,5] (see [6,7]). In ZSF interpretation it is expressed the well-known fact that the description of positive energy

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antiparticles moving forward in space and time is formally equivalent to the description of negative energy particles moving backward in space and time. In the BST this is also applied to the rest frames of negative energy particles, which also move backward in space and time, and in this more general form is formulated as *the equivalence principle of particle physics*.

When quantizing fields, it was generally accepted that ZSF interpretation leads to negative probabilities for states of negative energy (see [8]). For this reason, ZSF interpretation was considered incompatible with QFT. As a result, the formulation of QFT with positive energy particles and antiparticles, where vacuum is the lowest energy state, was accepted as standard [9,10]. In this formulation, the antiparticle operators were entered manually by replacing the creation (annihilation) operators of negative energy particles with the annihilation (creation) operators of positive energy antiparticles. However, the result of this artificial replacement was the appearance in the vacuum of diverging zero-point energy (and zero-point charge for charged fields), even when it was absent at negative energies, in particular for complex fields [2].

When solving practical problems, this problem, in fact a catastrophe, with the energy (and charge) of the vacuum was pragmatically ignored. For this purpose, the normal ordering recipe was postulated, an even more artificial operation that meant discarding divergent vacuum energy (and charge). However, the existence of gravity did not allow one to just ignore the predicted so large vacuum energy. As the result, the standard formulation of QFT, due to the obvious contradiction with general relativity, was in fact inconsistent. This fundamental problem, delicately called the cosmological constant problem, has also failed to be solved in almost all attempts to generalize QFT and the Standard Model (SM) of particle physics [11].

At the same time, when returning to the covariant formulation of QFT with negative energies, the problem with vacuum energy disappears in the special case of complex fields. In [2], exactly this approach was developed and the method of *time-symmetric quantization* (TSQ) was formulated, which ensured the compatibility of ZSF interpretation and quantum field theory. In TSQ, the probabilities of states are positive and there is no contribution of complex fields to the cosmological constant. In the present article it is shown that TSQ is a special case of TST and from this more general point of view, some aspects of TSQ are clarified.

In particular, it is shown that physical degrees of freedom of relativistic fields are described by complex field operators, which are always associated with corresponding probability current. If the degrees of freedom correspond to charged states, then the interaction current operator is given by the product of the probability current, existing for any field, and the matrix of interaction constants.

It is shown in the article that TSQ leads to a logically consecutive and physically consistent formulation of QFT without zero-point vacuum energy. In particular, the physical degrees of freedom of the photon field, other massless gauge fields and the graviton field, being transverse components of the field, are described as a complex scalar field with a chiral charge, the role of which plays helicity.

It is also shown that in TSQ, causal propagators and diagram technique are constructed naturally without former heuristic rules, and the final results coincide with the results of the standard diagram techniques. Thus, TSQ correctly describes the known observable effects.

The application of the TSQ to free scalar and fermion fields is described in Section 2. In Section 3, TSQ is applied to bosonic and fermionic fields, and in Section 4 to bosonic fields of SM. In Section 5 propagators, interacting fields, and diagram technique are considered. A more detailed description of TSQ is given in the book [12].

2. Time-symmetric quantization of relativistic fields

2.1. Relativistic fields in TST

In ZSF interpretation, the description of positive energy antiparticles moving forward in space and time is equivalent to the description of particles of negative energy moving backward in space and time. TST also introduces the latter's rest systems, consisting of negative energy particles, which also move backward in space and time.

As a result, the group of coordinate transformations of such frames of reference is extended, taking into account their doubling, and transitions between two classes of coordinate systems are given by 4-inversion. The latter includes changing the signs of 4-vectors, in particular $x_-^\mu = -x^\mu$, $p_-^\mu = -p^\mu$, $\mu = 0,1,2,3$, as well as actions on bispinors of the 4-inversion operator $PT = \gamma^5$ together with the Lorentz transformation operator S. Here 4-inversion operators PT, acting on the creation-annihilation operators of field quanta, will be considered also.

The energy k_0 of quanta of relativistic field is related to their 3-momentum $\mathbf{k}$ by the relation $k_{0\pm} = \pm\omega_k$, $\omega_k = \sqrt{\mathbf{k}^2 + m^2}$ and the complete set of solutions to the field equations includes states with both signs of energy. This doubles the Hilbert space of states of a quantum field, regardless of its tensor properties and spin of its quantum, one of which refers to positive energy particles, and the other to their antiparticles, described in terms of negative energy particles.

Complex wave functions of states, which in relativistic quantum mechanics satisfied the Klein-Gordon (KG), Dirac, etc. equation, now become field operators that satisfy the same equation.

For these two reasons, relativistic fields are described in TST by complex field functions.

2.2. Complex scalar and vector fields

After writing the backward in time integrals in terms of forward in time integrals, the action function for a complex scalar field $\phi(\mathbf{x},t)$ and its Hermitian conjugate component $\phi^+(\mathbf{x},t)$ takes the compact form:

$$S = \int d^4x\,(\partial_\mu\phi^+ \cdot \partial^\mu\phi - m^2\phi^+\phi)\ . \tag{1}$$

The corresponding Hamiltonian, probability current and canonical momenta are:

$$H = \int d^3x\,(\pi\pi^+ + \nabla\phi^+ \cdot \nabla\phi + m^2\phi^+\phi), \tag{2}$$

$$J_0 = i\int d^3x\,(\phi^+\pi^+ - \pi\phi), \tag{3}$$

$$\pi = \frac{\partial L}{\partial(\partial_t\phi)} = \partial_t\phi^+,\quad \pi^+ = \frac{\partial L}{\partial(\partial_t\phi^+)} = \partial_t\phi. \tag{4}$$

The field equations and equal time commutators have the form:

$$(\partial_\mu\partial^\mu - m^2)\phi = 0\ ,\quad (\partial_\mu\partial^\mu - m^2)\phi^+ = 0\ . \tag{5}$$

$$[\phi(\mathbf{x},t), \pi(\mathbf{x}',t)] = [\phi^+(\mathbf{x},t), \pi^+(\mathbf{x}',t)] = i\delta^3(\mathbf{x}-\mathbf{x}'). \tag{6}$$

The field operators and the operators of canonical momentum are further expanded in terms of solutions of the field equations (5):

$$\begin{aligned} \phi(x) &= \phi_+ + \phi_- = \int d\tilde{k}\,(a_k e^{-ikx} + a_{-k}e^{ikx}), \\ \phi^+(x) &= \phi_+^+ + \phi_-^+ = \int d\tilde{k}\,(a_k^+ e^{ikx} + a_{-k}^+ e^{-ikx}), \end{aligned} \tag{7}$$

$$\pi(x)=\pi_{+}+\pi_{-}=i\int d\tilde{k}\omega_{k}(a_{k}^{+}e^{ikx}-a_{-k}^{+}e^{-ikx}),$$
$$\pi^{+}(x)=\pi_{+}^{+}+\pi_{-}^{+}=-i\int d\tilde{k}\omega_{k}(a_{k}e^{-ikx}-a_{-k}e^{ikx}). \tag{8}$$

$$\int d\tilde{k}\equiv\int\frac{d^{2}k}{(2\pi)^{3}2\omega_{k}},$$

Substituting (7)-(8) into (6), we obtain commutators for the creation-annihilation operators of field quanta of both signs of energy, non-zero of which are:

$$[a_{k},a_{k'}^{+}]=2\omega_{k}(2\pi)^{3}\delta^{3}(\mathbf{k}-\mathbf{k}'), \tag{9}$$

$$[a_{-k},a_{-k'}^{+}]=-2\omega_{k}(2\pi)^{3}\delta^{3}(\mathbf{k}-\mathbf{k}'). \tag{10}$$

Consequences of the minus sign in (10) are considered below in Section 2.3. The observables of the field (2)-(3) are expressed in terms of these ladder operators as:

$$H=\int d\tilde{k}\,\omega_{k}(a_{k}^{+}a_{k}+a_{-k}^{+}a_{-k}), \tag{11}$$

$$J_{0}=\int d\tilde{k}(a_{k}^{+}a_{k}-a_{-k}^{+}a_{-k}). \tag{12}$$

The annihilation operators define the vacuum for both types of particles:

$$a_{k}\left|0\right\rangle=0,\quad a_{-k}\left|0\right\rangle=0, \tag{13}$$

in which, as can be seen from (11)-(12), there is no zero-point energy and zero-point probability current. Single-particle states are defined as:

$$a_{k}^{+}\left|0_{+}\right\rangle=\left|k\right\rangle,\quad a_{-k}^{+}\left|0_{-}\right\rangle=\left|-k\right\rangle. \tag{14}$$

Let us now consider the norm and amplitude of the probability of single-particle states with negative energy. Until now, it was generally accepted that the negative sign of the commutator in (10) leads to a negative norm of states in the Fock space. In fact, at writing (9)-(10) in general form:

$$\left\langle\pm k'\middle|\pm k\right\rangle=\left\langle 0_{\pm}\middle|a_{\pm k'}a_{\pm k}^{+}\middle|0_{\pm}\right\rangle=2k_{0\pm}(2\pi)^{3}\delta^{3}(\mathbf{k}-\mathbf{k}'), \tag{15}$$

it is clear that in this normalization, firstly, the sign of the commutator in (10) refers only to the sign the energy $k_{0\pm}=\pm\omega_{k}$ of the quanta, and secondly, this matrix element gives not the probability of the state, but the probability current of particles with this energy.

The wave function (probability amplitude) of a single-particle state $\psi_{-k'}(x)$ is given by the matrix element from the field operator (7) (see [9,10]):

$$\psi_{-k}(x)=\left\langle 0_{-}\middle|\phi(x)\middle|-k\right\rangle=\int d\tilde{k}'\left\langle 0_{-}\middle|a_{-k'}\middle|-k\right\rangle e^{ik'x}=$$
$$=\int d\tilde{k}'\left\langle 0_{-}\middle|a_{-k'}a_{-k'}^{+}\middle|0_{-}\right\rangle e^{ik'x}=-e^{ikx}. \tag{16}$$

The negative sign of the probability current in (15) changes the sign of the wave function in (16), but not the sign of the probability, which is given by the expression bilinear in ψ.

In this regard, it should be noted that the inclusion of energy of quanta into the normalization coefficients of wave functions, directly or through field operators, changes the physical meaning of the matrix elements. In the case of a scalar particle, the matrix elements, which in the non-relativistic theory gave probabilities, in the relativistic theory give probability currents, which have a different sign for particles moving in opposite directions of the time axis.

Thus, the negativity of the norm of a single-particle state (10) (and many-particle states with an odd number of particles) expresses only the fact that in TST this norm gives the probability current along the time axis, which in this case is directed inversely to this axis, while the probability of the state itself remains positive.

A massive complex vector field $B_\mu(x)$ contains the polarization vector $\varepsilon^\lambda_{\mu k}$ and its frequency expansion is [9,10]:

$$B_\mu(x) = \sum_\lambda \int d\tilde{k}(a_{k\lambda}\varepsilon^\lambda_{\mu k}e^{-ikx} + a_{-k\lambda}\varepsilon^{\lambda+}_{\mu k}e^{ikx}),$$
$$B^+_\mu(x) = \sum_\lambda \int d\tilde{k}(a^+_{k\lambda}\varepsilon^{\lambda+}_{\mu k}e^{ikx} + a^+_{-k\lambda}\varepsilon^\lambda_{\mu k}\, e^{-ikx}). \tag{17}$$

After separating three independent degrees of freedom, each of them is quantized in a given frame of reference independently as three complex scalar fields. Therefore, the Hamiltonian and the probability current of the vector field have the form:

$$H = \sum_\lambda \int d\tilde{k}\,\omega_k(a^+_{k\lambda}a_{k\lambda} + a^+_{-k\lambda}a_{-k\lambda}),$$
$$J_0 = \sum_\lambda \int d\tilde{k}(a^+_{k\lambda}a_{k\lambda} - a^+_{-k\lambda}a_{-k\lambda}). \tag{18}$$

They also do not contain zero-point energy and zero-point probability current of the vacuum.

This result about the absence of zero-point energy of the vacuum of a complex vector field, obtained in one frame of reference, is valid for all frames of reference due to the invariance of the vacuum.

2.3. Fermion field

In TST, the Dirac theory of fermion field does not change for the states of positive energy and spin ½, evolving forward in time in the ordinary inertial frame K_+. The fermion field ψ_+ is expanded into the wave functions of free positive energy particles with 4-momentum p^μ:

$$\psi_+(x) = 2m\sum_\alpha \int d\tilde{p}\, b_{p\alpha}u^\alpha_p e^{-ipx}, \qquad \psi^+_+(x) = 2m\sum_\alpha \int d\tilde{p}\, b^+_{p\alpha}u^{\alpha+}_p e^{ipx}. \tag{19}$$

Bispinors u^α_p are usually found in the rest frame of these particles K_+', and then the inverse Lorentz transformation is used to obtain their form in K_+, where the particles move [6,7].

In TST, the description of the negative energy states of fermion field, as in relativistic quantum mechanics [1], is significantly modified in comparison with the Dirac theory. These states, described by bispinors ψ^α_- in the inertial frame K_-, evolve backward in ordinary time, like K_- itself, and in this frame with coordinates x^μ_-, their momentum expansion is similar to (19):

$$\psi_-(x_-) = 2m\sum_\alpha \int d\tilde{p}\, b_{-p\alpha}u^\alpha_{-p}e^{-ip_-x_-}, \quad \psi^+_-(x_-) = 2m\sum_\alpha \int d\tilde{p}\, b^+_{-p\alpha}u^{\alpha+}_{-p}e^{ip_-x_-}. \tag{20}$$

Bispinors u^α_{-p} should now be found first in K_-', the rest frame of negative energy particles. Then, by an inverse Lorentz transformation, it will be possible to obtain their form in K_-, where these particles move, and only then, by a 4-inversion transformation, it will be possible to obtain their form in K_+. In this case, as will be shown below, the result will coincide with v_p^α, but due to the additional 4-inversion operation, $\bar{u}_{-p}^\alpha$ will transforms not to $\bar{v}_p^\alpha$, but to $-\bar{v}_p^\alpha$, which will change the sign of all bilinear forms where $\bar{v}_p^\alpha$ is present.

Notice that the former practice, when, using ZSF interpretation, an expression for v_p^α was found in the rest frame of a positive energy particle (see [6,7]), was erroneous, since particles with opposite signs of 4-momentum do not have a common rest frame.

In TST, as it was shown in [2], the action function for the Dirac field is reduced to the standard form ($\bar{\psi} = \psi^+\gamma^0$):

$$S = \int d^4x \left(\frac{i}{2}[\bar{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\bar{\psi})\gamma^\mu\psi] - m\bar{\psi}\psi \right). \tag{21}$$

This gives the Hamiltonian, the probability current and the Dirac equations:

$$H = \frac{i}{2}\int d^3x\,[\bar{\psi}\gamma^0(\partial_t\psi) - (\partial_t\bar{\psi})\gamma^0\psi], \tag{22}$$

$$J_0 = \int d^3x \bar{\psi}\gamma^0\psi, \tag{23}$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0, \quad \bar{\psi}(i\gamma^\mu\partial_\mu - m) = 0. \tag{24}$$

The frequency decomposition of the field operators in terms of solutions of (24) gives:

$$\begin{aligned} \psi(x) &= \psi_+ + \psi_- = 2m\sum_\alpha \int d\tilde{p}(b_{p\alpha}u_p^\alpha e^{-ipx} + b_{-p\alpha}v_p^\alpha e^{ipx}), \\ \psi^+(x) &= \psi_+^+ + \psi_-^+ = 2m\sum_\alpha \int d\tilde{p}(b_{p\alpha}^+ u_p^{\alpha+} e^{ipx} + b_{-p\alpha}^+ v_p^{\alpha+} e^{-ipx}). \end{aligned} \tag{25}$$

For bilinear forms of bispinors for positive-energy states, the standard relations hold ($E_p = (\mathbf{p}^2 + m^2)^{1/2}$):

$$\bar{u}_p^\alpha u_p^{\alpha'} = \delta^{\alpha\alpha'}, \quad \bar{u}_p^\alpha \gamma^0 u_p^{\alpha'} = \frac{E_p}{m}\delta^{\alpha\alpha'}, \quad \sum_\alpha u_p^\alpha \bar{u}_p^\alpha = \frac{\gamma p + m}{2m}. \tag{26}$$

In TST, for negative energy states evolving backward in time, in the frame K_-, also moving backward in time, the bilinear forms of the corresponding bispinors $u_{-p}^{\alpha'}$ are similar:

$$\bar{u}_{-p}^\alpha u_{-p}^{\alpha'} = \delta^{\alpha\alpha'}, \quad \bar{u}_{-p}^\alpha \gamma^0 u_{-p}^{\alpha'} = \frac{E_{-p}}{m}\delta^{\alpha\alpha'}, \quad \sum_\alpha u_{-p}^\alpha \bar{u}_{-p}^\alpha = \frac{\gamma p_{(-)} + m}{2m}. \tag{27}$$

When transforming these bispinors from K_- to K_+, including the Lorentz transformation with the matrix S, as well as 4-inversion, we obtain:

$$\begin{aligned} &u_{-p}^\alpha = \gamma^5 S v_p^\alpha, \quad \bar{u}_{-p}^\alpha = -\bar{v}_{-p}^\alpha S^{-1}\gamma^5 = \bar{\bar{v}}_p^\alpha S^{-1}\gamma^5 \\ &\bar{\bar{v}}_p^\alpha \equiv -\bar{v}_p^\alpha = -v_p^{\alpha+}\gamma^0. \end{aligned} \tag{28}$$

Therefore, bilinear forms for them take the form:

$$\bar{\bar{v}}_p^\alpha v_p^{\alpha'} = \delta^{\alpha\alpha'}, \quad \bar{\bar{v}}_p^\alpha \gamma^0 v_p^{\alpha'} = -\frac{E_p}{m}\delta^{\alpha\alpha'}, \quad \sum_\alpha v_p^\alpha \bar{\bar{v}}_p^\alpha = \frac{-\gamma p + m}{2m}, \tag{29}$$

since under the Lorentz transformation with 4-inversion the vectors are transformed with coefficients $-a^{\mu}_{\nu}$, which also gives:

$$S^{-1}\gamma^{\mu}S = -a^{\mu}_{\nu}\gamma^{\nu}. \tag{30}$$

They differ in sign from the corresponding expressions of the Dirac theory. Such changes in signs were previously compensated by changes in the signs of the anticommutators of birth-annihilation operators, which will be discussed below. In TST, the 4-components of the probability current $\overline{\overline{v}}^{\alpha}_{p}\gamma^{0}v^{\alpha'}_{p}$ are equal $-\delta^{\alpha\alpha'}E_p / m$, i.e. negative-definite, as it should be for the probability current of particles moving backward along the ordinary time axis.

Since the wave functions of fermions are antisymmetric, the field operators must change sign at rearrangement. Therefore, when field operators are rearranged in commutators, their product changes sign and the commutator turns into an anticommutator. As a result, the fermion field is quantized by simultaneous anticommutators ($\pi = i\psi^{+}$):

$$\left\{\psi^{\alpha}(\mathbf{x},t),\pi^{+\alpha'}(\mathbf{x}',t)\right\} = i\left\{\psi^{\alpha}(\mathbf{x},t),\psi^{+\alpha'}(\mathbf{x}',t)\right\} = i\delta^{3}(\mathbf{x}-\mathbf{x}')\delta^{\alpha\alpha'}, \tag{31}$$

$$\{b_{\pm p\alpha}, b^{+}_{\pm p'\alpha'}\} = \pm\frac{E_p}{m}\delta^{3}(\mathbf{p}-\mathbf{p}')\delta_{\alpha\alpha'}. \tag{32}$$

It is important here is that the negative sign in (32) appears due to a change in the sign of bilinear forms with conjugate spinors .

As the result, the expressions (22)-(32) give for observables:

$$H = \sum_{\alpha}\int d^{3}p\,\frac{m}{E_p}(b^{+}_{p\alpha}b_{p\alpha} + b^{+}_{-p\alpha}b_{-p\alpha})E_p\,, \tag{33}$$

$$J_0 = \sum_{\alpha}\int d^{3}p\,\frac{m}{E_p}(b^{+}_{p\alpha}b_{p\alpha} - b^{+}_{-p\alpha}b_{-p\alpha}). \tag{34}$$

Here the negative energy terms also differ in sign from the previous expressions of the Dirac theory.

Thus, in the vacuum of the fermion field, defined as $b_{\pm p\alpha}\left|0\right\rangle = 0$, there is also no zero-point energy and zero-point probability current. Since in TSQ there is no zero-point energy of the bosonic and fermionic relativistic fields, then in supersymmetric theories the breaking of supersymmetry does not lead to the appearance of the zero-point vacuum energy.

3. Standard model bosons and gravitons in TST

3.1. Massless gauge fields and graviton field

At considering the physical aspects of quantization of massless gauge fields and graviton field, as it was shown in [2], it is enough to quantize the physical components of the field in the source's rest frame, and then the results can be transformed to the required frame of reference.

Let us briefly consider the quantization of the photon field. At the condition $\partial_{\mu}A^{\mu} = 0$ the action function for the field in terms of the ordinary time integrals takes the form:

$$S = -\frac{1}{4}\int d^{4}x\; F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\int d^{4}x\; \partial_{\mu}A^{\nu}\cdot\partial^{\mu}A_{\nu}. \tag{35}$$

In the rest frame of the source $\mathbf{k}=(0,0,k^3)$ and the gauge $A_3=A_0=0$, the transverse physical components of circularly polarized photons form an analogue of the complex scalar field $A=(A_1+iA_2)/\sqrt{2}$, $A^+=(A_1-iA_2)/\sqrt{2}$. Then Lagrangian (35) and Hamiltonian, therefore, are analogous to the case of a complex scalar field:

$$L=\int d^3x\,\partial_\mu A^+\cdot\partial^\mu A, \tag{36}$$

$$H=\int d^3x\,(\pi\pi^+ +\nabla A^+\cdot\nabla A), \tag{37}$$

where $\pi(x)=\partial_t A^+$, $\pi^+(x)=\partial_t A$. The description of the field of circularly polarized photons in terms of complex variables diagonalizes the Hamiltonian (37) and helicity:

$$\Lambda=i\int d^3x\,(A^+\pi^+ -\pi A). \tag{38}$$

The latter is, in fact, a chiral charge, and photons with helicities $\Lambda=\pm 1$ are like a particle and a charge-conjugate antiparticle.

The field equations $\partial_\mu\partial^\mu A=0$, $\partial_\mu\partial^\mu A^+=0$ give the frequency expansion:

$$A=\int d\tilde{k}(a_k e^{-ikx}+a_{-k}e^{ikx}),\ \ \pi=i\int d\tilde{k}\omega_k(a_k^+e^{ikx}-a_{-k}^+e^{-ikx}), \tag{39}$$

where $a^+_{\pm k}, a_{\pm k}$ are the creation-annihilation operators for $\Lambda_+=+1$. For commutators, this gives:

$$[A(\mathbf{x},t),\pi(\mathbf{x}',t)]=i\delta^3(\mathbf{x}-\mathbf{x}'), \tag{40}$$

$$[a_{\pm k},a^+_{\pm k'}]=\pm(2\pi)^3 2\omega_k\delta^3(\mathbf{k}-\mathbf{k}'), \tag{41}$$

At an arbitrary orientation of the coordinate axes, a projection operator will appear here, selecting the transverse components. In Section 2.2 it was shown that the negative sign of the commutator in (41) does not change the sign of probabilities and they remain positive definite as in the case of a scalar field.

Operators of observables are:

$$\begin{aligned} H&=\int d\tilde{k}\,\omega_k(a_k^+a_k+a_{-k}^+a_{-k}),\\ \Lambda^0&=\int d\tilde{k}(a_k^+a_k-a_{-k}^+a_{-k}). \end{aligned} \tag{42}$$

This shows that the vacuums, defined as $a_{\pm k}|0\rangle=0$, do not contain zero-point energy and zero-point helicity.

Quanta of a massless non-Abelian vector gauge field A^a_μ (spin 1) and gravitons (spin 2) in the rest frame of the source, when the axis x^3 is directed along the momentum of the emitted quantum, also have two transverse physical degrees of freedom. There is axial symmetry here, and, for circularly polarized states, the free Hamiltonian and helicity are diagonal.

For this reason, their quantization is similar to the case of a photon field. Further consideration of nonlinearity, i.e. interactions of different field components, and internal symmetries does not change the conclusion about the absence of the zero-point energy of vacuum, which is part of the free Hamiltonian. The nonlinear contributions to the field energy are proportional to the coupling constants and decrease in the weak field limit, while the zero-point energy, as the energy of vacuum fluctuations of physical components in the free Hamiltonian, does not depend on the interaction constants.

The conclusion that when massless gauge fields and the graviton field are quantized in one frame of reference and in the transverse gauge, zero-point energy does not arise, remains valid for all frames of reference and gauges due to the invariance of the vacuum [12].

3.2. Bosons of electroweak theory

In [2], the quantization of bosonic fields of the electroweak theory was considered within the framework of TSQ, and here only the refinements introduced by TST as a more general theory will be considered.

If there is a particle-antiparticle pair with a non-zero mass, then in TST the antiparticle is described as a negative energy particle moving backward in time and in such a theory there is no zero-point vacuum energy. The field operators of two transverse components of the charged gauge field $W^{\pm}$ are mutually charge-conjugate and their quanta behave like a particle-antiparticle.

In the case of the components of a neutral gauge field and two neutral scalar fields, these fields, being electrically neutral, had a non-zero weak interaction charge. Therefore, before the interaction terms imparted mass to them, these pairs of components were mutually charge-conjugated. This leads to the fact that their free Hamiltonian did not contain zero energy, and the inclusion of interactions (even to generate an effective mass) no longer generates zero-point energy of the free field vacuum (details see in [2]).

From a more general point of view, there are no particles in TST that do not have an antiparticle, since any relativistic field is described by a complex field due to the presence of two signs of energy. The equality of the currently known quantum numbers of a particle and its antiparticle leads to their practical indistinguishability, but this does not mean their complete identity. There is always the possibility that there is some very weak interaction that causes them to be distinguishable.

Thus, at time-symmetric quantization of the fields of electroweak theory, zero-point energy and zero-point charge are absent in the case of massive fields also.

4. Commutators, time-symmetrical ordering and propagators

4.1. Commutators and time-symmetrical ordering of operators

The commutators for a complex scalar field are [2]:

$$[\phi(x'),\phi^*(x)] = D(x'-x), \tag{43}$$

$$[\phi(x'),\pi(x)] = i\partial_t D(x'-x), \tag{44}$$

where $D(x'-x)$ is the Pauli-Jordan function, which disappears behind the light cone. For a fermion field, the canonical momentum is equal to $\pi = i\psi^+$, and the analogue of (44) is:

$$\left\{\psi(x'),\bar{\psi}(x)\right\} = -(i\gamma^{\mu}\partial_{\mu} + m)iD(x'-x). \tag{45}$$

It is consistent with simultaneous anticommutators (32).

The chronological ordering operator T_+, on which the previous standard formulation of QFT was based, in TSQ acts only on operators of positive energy states, arranging them in order of *increasing* time from right to left:

$$T_+[A_+(t')B_+(t)] = \begin{cases} A_+(t')B_+(t), & t' > t, \\ B_+(t)A_+(t'), & t' < t. \end{cases} \tag{46}$$

In TSQ, an operator T_-, inverse to T_+, is additionally introduced acting only on operators of negative energy states and ordering them in the reverse order, placing them in order of *decreasing* time from right to left:

$$T_-[A_-(t')B_-(t)] = \begin{cases} B_-(t)A_-(t'), & t' > t, \\ A_-(t')B_-(t), & t' < t. \end{cases} \tag{47}$$

In the general case, the time-symmetric ordering operator $\hat{T}$ is introduced in TSQ as the product of the previous two operators. For each class of field operators, it is reduced to their chronological ordering operators: $\hat{T} = T_+T_- = T_-T_+$, acting on the operators of both signs of energy selectively and ordering in the reverse order. In particular, when $t' > t$ we have:

$$\hat{T}[A_+(t')B_+(t)A_-(t')B_-(t)] = [A_+(t')B_+(t)][B_-(t)A_-(t')]. \tag{48}$$

4.2. Causal propagators of fields

In TSQ, causal propagators for bosons $G_c(x-x')$ and fermions $S_c(x-x')$ appear naturally, as was shown in [2]. Here we will make this conclusion with clarifications following from TSQ as a more general theory.

For a scalar field G_c is a Green's function under the boundary conditions of ZSF, corresponding to the propagation forward in time of solutions with positive energy and backward in time with negative energy.

For the Klein-Gordon equation with a source $j(x)$, the propagators $G_{c\pm}(x'-x)$ are defined for solutions of two energy signs as:

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = j(x) \tag{49}$$

$$(\partial_\mu \partial^\mu + m^2)G_{c\pm}(x'-x) = \delta^4(x'-x). \tag{50}$$

The solution to field equations (49), taking into account translational invariance and boundary conditions of ZSF, has the form in terms of ordinary time:

$$\phi(x') = \phi^{(0)}(x') + \int_{-\infty}^{\infty} dt \int d^3x\, G_{c+}(x'-x)\, j(x) + \int_{\infty}^{-\infty} dt \int d^3x\, G_{c-}(x'-x)\, j(x). \tag{51}$$

Here $\phi^{(0)}$ is a solution to a homogeneous equation with the same boundary conditions, $G_{c+}(x'-x) \sim \theta(t'-t)$ describes the transfer of positive energy forward in time, and $G_{c-}(x'-x) \sim \theta(t-t')$ describes the transfer of negative energy backward in time.

The limits of the second time integral in (51) are rearranged (transition from the future to the past) and the integrals of the two terms cannot simply be combined. In order to write down a complete causal propagator G_c, the limits of the second integral must be inverted, which gives a minus sign in front of G_{c-}, and (51) takes a more compact form:

$$\begin{aligned} \phi(x) &= \phi^{(0)}(x) + \int_{-\infty}^{\infty} dt' \int d^3x' [G_{c+}(x-x') - G_{c-}(x-x')]\, j(x') = \\ &= \phi^{(0)}(x) + \int_{-\infty}^{\infty} dt' \int d^3x'\, G_c(x-x')\, j(x') = \phi^{(0)}(x) + \int d^4x'\, G_c(x-x')\, j(x'), \end{aligned} \tag{52}$$

where

$$G_c(x) = G_{c+}(x) - G_{c-}(x). \tag{53}$$

Thus, when expressing the inverse time integral through the integral in the forward direction, the causal propagator G_c is given by *the difference* of G_{c+} and G_{c-}.

Causal propagators $G_{c\pm}$ for a complex scalar field are defined as chronological products of field operators for two signs of energy:

$$iD_{c\pm}(x'-x) = \langle 0 | T_\pm[\phi_\pm(x')\phi_\pm^*(x)] | 0 \rangle. \tag{54}$$

Substituting frequency expansions gives:

$$\begin{aligned} iD_{c+}(x'-x) &= \langle 0 | T_+[\phi_+(x')\phi_+^*(x)] | 0 \rangle = \\ &= \langle 0 | \phi_+(x')\phi_+^*(x) | 0 \rangle \theta(t'-t) + \langle 0 | \phi_+^*(x)\phi_+(x') | 0 \rangle \theta(t-t') = \\ &= \int d\tilde{k} d\tilde{k}' [\langle 0 | a_{k'} a_k^* | 0 \rangle \theta(t'-t) + \langle 0 | a_k^* a_{k'} | 0 \rangle \theta(t-t')] e^{-i(k'x'-kx)} = \\ &= \int d\tilde{k}\, \theta(t'-t) e^{-ik(x'-x)}. \end{aligned} \tag{55}$$

Here the matrix element $\langle 0 | \phi_+^*(x)\phi_+(x') | 0 \rangle$ disappears and only the contribution of $\langle 0 | \phi_+(x')\phi_+^*(x) | 0 \rangle$ remains. For the negative energy it is similar:

$$\begin{aligned} iD_{c-}(x'-x) &= \langle 0 | T_-[\phi_-(x')\phi_-^*(x)] | 0 \rangle = \\ &= \langle 0 | \phi_-^*(x)\phi_-(x') | 0 \rangle \theta(t'-t) + \langle 0 | \phi_-(x')\phi_-^*(x) | 0 \rangle \theta(t-t') = \\ &= \int d\tilde{k} d\tilde{k}' [\langle 0 | a_{-k}^* a_{-k'} | 0 \rangle \theta(t'-t) + \langle 0 | a_{-k'} a_{-k}^* | 0 \rangle \theta(t-t')] e^{i(k'x'-kx)} = \\ &= -\int d\tilde{k}\, \theta(t-t') e^{ik(x'-x)}. \end{aligned} \tag{56}$$

In the future, it is convenient to convert the integrals back over time into ordinary integrals over time, which changes the sign, but allows you to write the formulas in a compact form. It is also convenient not to indicate this change of sign explicitly, and under integrals in ordinary time and with ordinary limits, one can redefine the operator T_- as also changing the sign of the product of operators, i.e. redefine (47) as:

$$T_-[A_-(t')B_-(t)] = \begin{cases} -B_-(t)A_-(t'), & t' > t, \\ -A_-(t')B_-(t), & t' < t. \end{cases} \tag{57}$$

The complete causal propagator in (52)-(53) thereby takes the standard form:

$$\begin{aligned} iD_c(x'-x) &= \langle 0 | \hat{T}\phi(x')\phi^*(x) | 0 \rangle \\ &= \langle 0 | T_+[\phi_+(x')\phi_+^*(x)] + T_-[\phi_-(x')\phi_-^*(x)] | 0 \rangle = \\ &= \langle 0 | \phi_+(x')\phi_+^*(x) | 0 \rangle \theta(t'-t) - \langle 0 | \phi_-(x')\phi_-^*(x) | 0 \rangle \theta(t-t') = \\ &= \int d\tilde{k} [\theta(t'-t) e^{-ik(x'-x)} + \theta(t-t') e^{ik(x'-x)}]. \end{aligned} \tag{58}$$

The standard result (58) also leads to the standard momentum representation:

$$iD_c(x'-x) = \frac{1}{(2\pi)^4}\int d^k k \frac{e^{-ik(x'-x)}}{k^2 - m^2 + i\varepsilon}. \tag{59}$$

The causal propagators for fermions $S_{c\pm}(x)$ can be obtained in a similar way by considering the chronological products of field operators in two directions of time:

$$\begin{aligned} iS_c(x'-x) &= \langle 0|T_+[\psi_+(x')\bar{\psi}_+(x)]|0\rangle + \langle 0|T_-[\psi_-(x')\bar{\psi}_-(x)]|0\rangle = \\ &= \langle 0|\psi_+(x')\bar{\psi}_+(x)|0\rangle\theta(t'-t) - \langle 0|\psi_-(x')\bar{\psi}_-(x)|0\rangle\theta(t-t') = \\ &= (2m)^2\sum_{\alpha'\alpha}\int d\tilde{p}d\tilde{p}'[\theta(t'-t)\langle 0|b_{p'\alpha'}b^+_{p\alpha}|0\rangle u^{\alpha'}_{p'}\bar{u}^{\alpha}_{p}e^{-i(p'x'-px)} - \\ &-\theta(t-t')\langle 0|b_{-p'\alpha'}b^+_{-p\alpha}|0\rangle v^{\alpha'}_{p'}\bar{v}^{\alpha}_{p}e^{i(p'x'-px)})], \end{aligned} \tag{60}$$

and finally:

$$\begin{aligned} &iS_c(x'-x) = \\ &= \int d\tilde{p}\sum_p\left[\theta(t'-t)(\gamma p + m)e^{-i\omega_p(t'-t)} + \theta(t-t')(-\gamma p + m)e^{i\omega_p(t'-t)}\right]e^{i\mathbf{p}(\mathbf{x}'-\mathbf{x})} = \\ &= -(i\gamma\partial + m)D_c(x'-x). \end{aligned} \tag{61}$$

Thus, in TSQ, the time-symmetric ordering of operators, performed by $\hat{T}$ (taking into account the change in sign at combining of time integrations) leads to the standard causal propagators.

5. Conclusion

Time-symmetric QFT, based on TSQ, is a consistent quantum theory of relativistic fields in a compact and covariant form. Relativistic fields, due to the doubling of the space of states, in TSQ are described by complex field functions regardless of the presence of a charge of any interaction.

The probability current (density) operator, which changes sign when the direction of evolution changes along the time axis, is nonzero for all quanta, including truly neutral ones. It transforms into the interaction current operator only when multiplied by the matrix of interaction constants. Thus, the interaction current (and the charge as its 4-component) disappears not because the probability current disappears, but because the interaction constant is equal to zero.

Time integration in action functions and in perturbation theory occurs in both directions of time depending on the sign of the energy. Therefore, the time-symmetric ordering operator $\hat{T} = T_+T_-$ is introduced in TSQ, which naturally leads to the causal propagators of ZSF.

This difference leads to a modification of the diagram technique, but when all expressions are written in terms of ordinary time with ordinary limits, which makes the formulas more compact, the results of the standard diagram technique are reproduced. Thus, TSQ correctly describes known observational effects.

It is shown that in TSQ there is no zero energy and zero vacuum charge of free fields and therefore this fact mainly eliminates the cosmological constant problem. TSQ, which excludes the zero-point vacuum energy of SM fields, is in agreement with known experiments, the results of which, until that time attributed to the zero-point energy, are in fact completely explained by the contributions of the fields of real charges.

A more detailed discussion of TSQ and its consequences is given in the book [12].

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