

Local and global theories of relativity in flat and curved spacetimes

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Abstract

Special and general theories of relativity consist in describing both local and global phenomena - the first in flat, and the second in curved spacetime. In the paper it is shown that each of these two classes of relativistic effects, local and global, is universal and is the subject of a separate theory. First, descriptions in local frames of reference, related by the local Lorentz transformations, form the local theory of relativity, or local relativity (LR). The locality principle allows to apply LR to non-inertial local frames, and the equivalence principle to the local frames in gravitational field. Secondly, descriptions in global frames of reference, constructed from local frames coexisting on a common hypersurface of simultaneity, form the global theory of relativity, or global relativity (GIR). LR and GIR are based on physical coordinates and complement each other, the special and general theories of relativity were hybrids of these two theories. LR and GIR describe the local and global properties of gravity, separating the field effects from the effects of motion by different methods, such as bimetric formalism, where one metric describes geometry of the global frames, and other describes spacetime geometry. It is shown that GIR leads to a picture of collapse with formation of frozars, and also leads to a cutoff of the loop integrals of quantum fields at the Planck length. In GIR, cosmological models are built on hypersurfaces of simultaneity, where both stretching and the Doppler effect contribute to redshifts, and aberration is also taken into account. Predicted an initial violetshift removing the double redshift paradox, and this leads to the slowing time cosmology consistent with observational data.

Keywords: Lorentz transformations, relativistic effects, non-inertial systems, gravity, spacetime curvature, locality principle, equivalence principle, quantum fields in gravitational field, gravitational collapse, cosmology.

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Introduction

Physical phenomena in space free of gravity take the simplest form in inertial frames of reference (IFR) and they are described by special relativity (SR), which is well confirmed [1]. The phenomena in non-inertial frames of reference (NFR) and in the gravitational field are described by general relativity (GR), but only part of the formalism of GR, associated with physical variables in the permissible range of their variation, has been experimentally confirmed [2].

GR, being formulated in an excessively general form, also included non-physical elements, such as arbitrary coordinates, and therefore, at applying GR to real systems with

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physical coordinates, it was always necessary to clarify which part of it describes real events and which does not [2]. In simple cases this is not difficult, but under extreme conditions the basic concepts of the theory had to be reconciled with the physical constraints of SR in the vicinity of each local frame. But, since this was usually not done, naively trusting all the consequences of the formalism of GR, non-physical conclusions were often made at describing phenomena in strong fields. Therefore, the problem of separating the physical part of GR from the nonphysical was not just a methodological, but a fundamental problem, the solution of which was to transform the physical part of GR into a consistent physical theory, all the consequences of which could be fully trusted.

The physical part of GR is, first of all, the consequences of the equivalence principle, according to which the SR is locally valid in gravitational fields. The formalism of GR, however, was interpreted on the basis of the principle of general covariance allowing arbitrary extended frames of reference, some of which could be connected by the local non-Lorentz transformations and contains the local parts existing non-simultaneously.

In the paper, the physical part of GR is formulated in the form of two independent parts. The first one is a description in local frames of reference (LFRs) connected by the local Lorentz transformations, and the second part is a description in global frames of reference (GFRs) built from LFRs coexisting on a common hypersurface of simultaneity. Since SR has the same structure, then two similar parts of SR and GR are combined into two separate theories.

At first, the description of local phenomena in LFRs, due to their universality, i.e. applicability for both inertial and non-inertial frames in both flat and curved spacetimes, is formulated as an independent theory - the local theory of relativity, or local relativity (LR). Secondly, the description of extended objects by means of GFRs also turned out to be universal, applicable for both inertial and non-inertial frames in both flat and curved spacetimes, also is formulated as a separate theory, the global theory of relativity, or global relativity (GIR) [3].

As a result, the local parts of SR and GR constituted the LR, and their global parts formed the GIR, i.e. fragments of SR and GR were redistributed, forming two universal theories, each of which describes physical phenomena in a unified way for both inertial and non-inertial frames in flat and curved spacetimes. At the same time, they complement each other, occupying an intermediate position between SR and GR. The first one, LR, is the result of the localization of SR, taking into account the experience of such localization in GR, and the second one, GIR, is the result of the consecutive globalization of LR with the preservation of such a key global aspect of SR as representation of physical reality, or the world, as a set of simultaneous events.

Classical mechanics at one time was also reformulated without changing its physical foundations - from the form of Newtonian mechanics it took on more universal forms of Lagrangian and Hamiltonian mechanics, which mutually complemented each other. The GIR explains the advantages of the Hamiltonian form in many areas of physics by the fact that the time variable of the Hamiltonian formulations has the meaning of world time expressing the global simultaneity of events in the coexisting parts of the GFR.

In part 1 of the paper, LR is formulated as a theory describing the phenomena in the vicinity of local frames. In part 2, GIR is formulated as a theory describing phenomena in extended frames on hypersurfaces of simultaneity. In part 3 the evolution of extended objects, in particular, quantum fields and stars, in strong gravitational fields is considered, cosmological consequences of the GIR are also presented. A more detailed description of LR and GIR, including their applications, will be presented in the book [4].

List of abbreviations: FR - frame of reference, LFR - local FR, GFR - global FR, IFR - inertial FR, NFR - non-inertial FR, SR - special relativity, GR - general relativity, LR - local relativity, GIR - global relativity.

1. Local relativity as a description of events in local frames of reference

A. Physics in inertial LFRs.

Physical phenomena in flat spacetime are described by SR by introducing global IFRs, but to describe events in the local vicinity of a particle, local-inertial frames or inertial LFR are sufficient. Further, the SR limited by LFRs will be called LR, which, as will be shown below, becomes because of this limitation a more universal theory than the SR itself, since LR is also applicable for non-inertial LFRs and LFRs in a gravitational field.

In the vicinity of any point in space, inertial LFRs can be introduced, the relative speeds of which may differ in magnitude and direction. Each of them consists of a basis in the form of a test particle of small mass, to which a local tetrad is attached. A tetrad has four mutually orthogonal unit vectors in spacetime \mathbf{e}_i ($i=0,1,2,3$), which from a physical point of view are three standard scales, forming a spatial triad, and a standard clock. The coordinates of the vector x^i in spacetime are entered as $\mathbf{x} = x^i \mathbf{e}_i$ (with the summation over the repeated index), where $x^0 = ct$. An event, a physical phenomenon at a given point in space at a given moment in time, is a world point in spacetime, and a continuous chain of events is a worldline.

Classical mechanics is based on *the principle of relativity*, according to which physical laws have the same form in all LFRs in a given place. By denoting the speed of the LFR K' relative to the LFR K along the x axis as V , one finds that the intervals of coordinates of two events in two LFRs are related by the Galilean transformations: $dt' = dt$ and $dx = dx' - vdt'$.

Experiments have shown the need to supplement classical mechanics with the principle of constancy of the speed of light in all LFRs [1]. This leads to the presence of invariant (light-like) isotropic lines in spacetime:

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = 0, \quad ds'^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 = 0, \quad (1)$$

The transition from one LFR to another, moving relative to the first, then occurs in the form of a rotation of the spatial and temporal coordinate axes towards each other. As a result, the time axis acts not only as the fourth coordinate, but makes space-time 4-dimensional pseudo-Euclidean space. Of two forms of describing such a space-time, 1) $\mathbf{e}_0^2 = -1$, $\mathbf{e}_a^2 = 1$, and 2) $\mathbf{e}_0^2 = 1$, $\mathbf{e}_a^2 = -1$, where $a = 1, 2, 3$, the second turns out to be convenient in practice, which will be used below.

Both events and vectors connecting two events are invariants of spacetime. The invariant vector $d\mathbf{x}$ of the interval between close events can be represented in terms of the coordinates of two LFRs as $d\mathbf{x} = dx^i \mathbf{e}_i = dx'^i \mathbf{e}'_i$. Taking into account the orthonormality of the basis vectors in both LFR:

$$\mathbf{e}_0^2 = \mathbf{e}'_0{}^2 = -1, \quad \mathbf{e}_1^2 = \mathbf{e}'_1{}^2 = 1, \quad \mathbf{e}_0 \cdot \mathbf{e}_1 = \mathbf{e}'_0 \cdot \mathbf{e}'_1 = 0, \quad (2)$$

for the square of the modulus $d\mathbf{x}^2$, usually denoted as ds^2 , one obtains in two LFRs:

$$d\mathbf{x}^2 \equiv ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2 = \eta_{ik} dx^i dx^k, \quad (3)$$

where $\eta_{ik} = \text{diag}(1, -1, -1, -1)$, $i, k = 0, \dots, 3$. Rotation by an angle χ on the pseudo-Euclidean plane (ct, x) is given by the formulas:

$$c\Delta t = \Delta x' \sinh \chi + c\Delta t' \cosh \chi, \quad \Delta x = \Delta x' \cosh \chi + c\Delta t' \sinh \chi. \quad (4)$$

For the origin of coordinates of the LFR K' $\Delta x' = 0$ and Eqs. (4) in this case give $c\Delta t = c\Delta t' \cosh \chi$, $\Delta x = c\Delta t' \sinh \chi$. This gives the expression for the rotation angle χ and coefficients in (4) in terms of the relative speed V :

$$\tanh \chi = \frac{\Delta x}{c \Delta t} = \frac{V}{c}, \quad \sinh \chi = \frac{V/c}{\sqrt{1-V^2/c^2}}, \quad \cosh \chi = \frac{1}{\sqrt{1-V^2/c^2}}. \quad (5)$$

As a result, the intervals between the coordinates and times of two close events are related in two LFRs by the Lorentz transformations:

$$dt' = \frac{dt - Vdx/c^2}{\sqrt{1-V^2/c^2}}, \quad dx' = \frac{dx - Vdt}{\sqrt{1-V^2/c^2}}, \quad dy' = dy, \quad dz' = dz. \quad (6)$$

Formulas expressing (t, x) through (t', x') are the same as (6), but with replacement $V \rightarrow -V$.

For the velocities of a particle in two LFRs $v = dx/dt$ and $v' = dx'/dt'$ we obtain from (6) the velocity addition law, according to which relative velocities does not exceed c :

$$v = \frac{v' + V}{1 + v'V/c^2} < c, \quad (7)$$

Time intervals are measured using events at the same place. Distance measurement, i.e. their comparison with the standard of length, is made by simultaneously fixing the ends of the scales, i.e. by fixing simultaneous events. But, since in (6) $dt' \neq dt$, different LFRs have different hypersurfaces of simultaneity, which means the relativity of simultaneity of events. This leads to relativistic effects - slowing down of proper times $d\tau$ and contracting the lengths of the standard scales for moving objects in comparison with the rate of time and lengths in their rest frame. The interval of the proper time $d\tau$ of a particle of mass $m > 0$, defined as:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\mathbf{r}^2 > 0, \quad d\tau = dt \sqrt{1 - V^2/c^2} \quad (8)$$

is invariant, and real intervals $ds^2 > 0$ are timelike. The proper length dl_0 , defined as:

$$ds^2 = -dl^2 = c^2 dt^2 - d\mathbf{r}^2 < 0, \quad d\tau = dt \sqrt{1 - V^2/c^2}, \quad dl = dl_0 \sqrt{1 - V^2/c^2}. \quad (9)$$

also is invariant and the imaginary intervals $ds^2 < 0$ are spacelike. The property of intervals to be lightlike ($ds^2 = 0$), timelike, or spacelike is invariant and does not depend on the LFR.

The first and most widely used application of LR is the case of inertial LFRs in flat spacetime without gravity, when these LFRs are part of the global IFR.

The evolution in time of particles and fields is given by the equations of motion, which follow from the principle of least action. For a free particle, it takes the form:

$$S = -mc \int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} L dt, \quad L = mc^2 \sqrt{1 - v^2/c^2}. \quad (10)$$

In the general case, the Lagrange function L can refer to any physical system in the vicinity of the LFR. Then equations of motion or equations of fields follow from it. 4-speed u^i has properties:

$$u^i = \frac{dx^i}{ds}, \quad u^0 = \frac{c}{\sqrt{1 - v^2/c^2}}, \quad \mathbf{u} = \frac{\mathbf{v}}{\sqrt{1 - v^2/c^2}}, \quad \eta_{ij} u^i u^j = -1. \quad (11)$$

Accordingly, the 4-momentum of the particle is $p^i = mu^i$, the components of which are energy and momentum $(E/c, \mathbf{p})$, while the rest energy is equal to $E_0 = mc^2$.

In the definition of the action function in (10) $L > 0$ at $t_2 > t_1$, but the sequence of times and the sign of L in the general case are not defined. Here the action function does not change

at a simultaneous change in the signs L and the integral (by permutation of the limits), which means $L < 0$ at $t_2 < t_1$. This property underlies the Stueckelberg-Feynman treatment, where antiparticles are described as the same particles, but with negative energy and going backward in time. To pass to such a treatment, the direction of evolution in time in (10) will be fixed by introducing a step function defined as $\theta(x) = 1$ and $\theta(-x) = 0$ at $x > 0$. Further, in (10), we rearrange the limits of the integral with respect to time, which will change the sign of the integral, but we assign this sign to L as $L \rightarrow -L$, and then, renaming the moments of time in the second integral, we obtain the action function for particles of negative energy going backward in time:

$$S = \theta(t_2 - t_1) \int_{t_1}^{t_2} L dt = \theta(t_2 - t_1) \int_{t_2}^{t_1} (-L) dt = \theta(t_1 - t_2) \int_{t_1}^{t_2} (-L) dt. \quad (12)$$

Introducing the step function in (10) also, we arrive at the general expression for particles and their antiparticles of positive energy, going only forward in time, and then we replace antiparticles with particles of negative energy going backward in time:

$$S = \theta(t_2 - t_1) \left(\int_{t_1}^{t_2} L dt + \int_{t_1}^{t_2} L dt \right) = \theta(t_2 - t_1) \int_{t_1}^{t_2} L dt + \theta(t_1 - t_2) \int_{t_1}^{t_2} (-L) dt. \quad (13)$$

This is the expression for the action function in the Stueckelberg-Feynman treatment [7], used in quantum field theory for the joint description of particles and antiparticles.

Thus, in LR the principle of relativity takes the form of the local Lorentz invariance. How this is achieved under specific physical conditions will be discussed below.

B. Physics in non-inertial LFRs. The locality principle.

In flat spacetime without gravity, there are non-inertial LFRs, which in the first approximation are also described by the LR. This follows from the fact that in this approximation the acceleration \mathbf{a} does not locally affect the rate of time and the lengths of the scales, which means that relativistic effects, as well as for inertial LFRs, are determined by relative velocities only (the property of locality). For weak accelerations, the property of locality takes place because changes in the time rate and the length of the scales can depend only on an even degree of acceleration \mathbf{a}^2 , and they can be neglected, since for a small interval Δt (locality in time) their contribution to time dilation will be of the order of smallness Δt^2 .

At strong accelerations, there are no such direct arguments for the locality property, but experiments do not contradict it and therefore, generalizing known experiments, it is called *the locality principle* [2]. An indirect argument is that there is an analogue in the gravitational field - *the equivalence principle*, which has been well confirmed in experiments (see below). Taking into account these circumstances, the locality principle will be considered valid in all non-inertial LFRs.

In this case, two non-inertial LFRs in the vicinity of the same event are connected in the first approximation by the local Lorentz transformations and the local Lorentz invariance takes place in them. To simplify the situation, comoving inertial LFRs are often introduced when the local Lorentz invariance is obvious. For them locality also means in time, i.e. during the time of comoving.

Thus, due to the locality principle, the LR is also valid in the first approximation in non-inertial LFRs, which makes it possible to describe local phenomena in such FRs by the same relations as in inertial LFRs.

C. Physics in LFR in a gravitational field. The equivalence principle.

Phenomena in an LFR at rest in a gravitational field occur similarly as in a non-inertial LFR moving with an acceleration of the same magnitude, but directed opposite to the gravitational acceleration. This fact, confirmed by experiments, lies on the basis of the equivalence principle [2].

From the equivalence principle, it follows that for LFR in a gravitational field, as in non-inertial LFRs, the local Lorentz invariance is valid in the first approximation, and therefore LR is valid in the same approximation. This makes it possible to describe local phenomena in the gravitational field by the same relations as in inertial LFR.

D. LR as a universal theory and transition to the global relativity

Thus, although LR is only a localized form of SR, nevertheless, unlike SR as a whole, it is applicable both in unaccelerated and accelerated LFRs, both in flat and in curved spacetimes. The point is that SR consisted of two parts - the local part, which is universal and therefore can appear as a separate theory, LTR, and the global part, which is limited only by inertial FRs in flat spacetime.

In the next Section, it will be shown that this global part of SR also becomes a part of another universal theory, GIR, applicable both for extended NFRs and for GFRs in a gravitational field. GIR, in contrast to LR, deals with more complicated systems, its formalism is more cumbersome and for this reason only the main statements and results will be given below, and a more details will be presented in subsequent publications and in the book [4].

2. Global relativity as a description of events on hypersurfaces of simultaneity

A. Physics in global IFR in flat spacetime, GIR in the form of SR.

In the LR, in the vicinity of each point in space, families of LFRs are introduced and local events are described in terms of local spatial coordinates and local proper times of one of the LFRs at a given place. This means that within the framework of LR, at point A, the "A-time" of one of the LFR is set, and at point B, which is outside the close vicinity of A, the "B-time" of another LFR is specified, but the common "time" for A and B is not defined [1]. The same is the case with the definition of the lengths of the standard scales.

Consider, therefore, the definitions of times and lengths common to the two LFR in A and B. To do this, at one point selected as the origin of coordinates, at the initial moment of time, i.e. in the event O , we will *select one of the LFRs at this point as the basic one*, and then from each family of LFRs at other points we will select one of LFRs at a time, simultaneous with the event O from the point of view of the observer in the basic LFR. At other times, we will repeat the same procedure. If at each moment of time in such a set of LFRs, simultaneously coexisting at different points, all neighboring standard clocks will be mutually synchronized (by Einstein synchronization), and neighboring spatial triads, consisting of standard scales, are continuously connected and numbered, then this construction forms the GFR. Even if the synchronization is broken after a short time, at least this period of time we deal with the GFR, and at other times we can repeat the same procedure, restoring the GFR in its initial form.

In this case, the physical coordinate system in space is formed by using the symmetry properties of the physical system describing by this GFR. Space without taking into account the influence of fields and particles is homogeneous and isotropic, and these symmetries of space are expressed by the Cartesian coordinate system. But, if a physical system has spherical symmetry, then a spherical system is constructed, etc.

By introducing the GFR, we thus go beyond the LR and the theory describing events by introducing the GFR, further we will call the GIR. In GIR physical reality appears in each of

the GFR as a set of particles and fields instantly coexisting on the hypersurfaces of simultaneity of the basic LFR.

An event, i.e. the world point O , can be chosen as the origin of coordinates from which the light cones depart into the past and the future. Events in and on these light cones are in the absolute past or absolute future relative to the present event O . Events outside the two branches of the light cone occur at a different point in space and are absolutely distant from O . Relativistic causality is that two events can be causally related only if the interval between them is timelike (inside a light cone) or lightlike. For such events, the concepts of "earlier" and "later" have an absolute meaning and this allows us to unambiguously define the concepts of cause and consequence.

The first and most frequently used application of the GIR is the case of flat space without gravity, when a global IFR is formed from LFR. Inertial LFR in different places of the global IFR are mutually resting and therefore for the locality of LFR it is enough to be limited to a small area of space, and a time limit is not necessary. In this case, the principle of relativity and the principle of constancy of the speed of light are valid in a global form, i.e. the laws of physics, including the constancy of the speed of light, are the same in all global IFRs. Thus, GIR in this simplest case of IFR in a flat world is reduced to SR. The principle of relativity here takes the form of *the global Lorentz invariance*. The homogeneity of space and time leads to translational invariance, which means the addition of initial values x_0^i for the coordinates of the basic inertial LFR K (or K'), after which the Poincaré group becomes the symmetry group:

$$x^{i'} = \Lambda_k^i x^k + x_0^i. \quad (14)$$

In LR, slowing down of times and contracting of lengths in moving local IFRs were the results of projection onto the hypersurface of the simultaneity of a resting LFR. In the global IFR, the situation is the same, but now globally.

The considered global construction of SR, GFR on the hypersurface of simultaneity of the basic LFR, turns out to be more universal than SR itself. Being applicable both for non-inertial GFRs, and for GFRs in a gravitational field, and this construction underlies the GIR, the theory which is as universal as the LR.

B. Physics in extended NFRs in flat spacetime.

In a flat spacetime, an extended *non-inertial* FR (NFR) can be formed from a set of non-inertial LFRs. On the one hand, such LFRs, forming an extended NFR, will not have a common hypersurface of simultaneity. But, on the other hand, the conditions of causality include the requirement that an extended NFR, as a unified system of local objects, should include only such LFRs that coexist simultaneously.

In a flat spacetime, this requirement of causality, about the simultaneous coexistence of LFRs of an NFR, is fulfilled at constructing the extended NFR on the hypersurface of simultaneity of one of the global IFRs, chosen as the basic one [5]. The spacetime interval between two close events, measured by a standard clock and the rods of the basic IFR, has the form:

$$ds^2 = c^2 dt^2 - dx^a dx_a = \gamma_{ik} dx^i dx^k. \quad (15)$$

Here the "flat" metric $\gamma_{ik} = (1, \gamma_{ab})$ of the basic IFR includes the 3-metric γ_{ab} of curvilinear coordinates and leads to a zero curvature tensor, which means that this IFR adequately expresses the geometry of flat spacetime.

If one of the two standard clocks is accelerated, then when they meet again, the accelerated clock will slow down w.r.t. the unaccelerated one absolutely, since the spacetime interval along the indirect worldline is less than that of the straight worldline. Particular cases of such absolute time dilation are oscillations and fluctuations of particles, when their proper times slow down irreversibly in comparison with the time of the rest frame of the center of

inertia of the system. Moreover, any time dilation with even short-term acceleration introduces an element of irreversibility. Therefore, non-inertial time dilation, which is absolute, like acceleration, cannot be confused with inertial time dilation, which is relative.

Further, we will take into account that for each of the non-inertial LFR in the extended NFR, the LR is locally valid, which follows from the locality principle. Therefore, the elements of extended NFRs are only such LFRs that are related by the local Lorentz transformations and translations. The interval measured by standard clocks and rods of an LFR of an extended NFR will be similar to (15):

$$ds^2 = c^2 dt^2 - dx^{a'} dx_{a'}. \quad (16)$$

The next step is projecting the spatial axes of all LFRs onto the hypersurface $t = \text{const.}$ of the basic IFR. At the same time, the standard scales and clocks of this IFR are considered to be the reference, expressing through them the standard scales and clocks of the NFR. The relationship between their small intervals is linear:

$$cdt = e_0^{0'} cdt + e_a^{0'} dx^a, \quad dx^{a'} = e_a^{a'} dx^a + e_0^{a'} cdt, \quad (17)$$

and the spacetime interval (16) is expressed in the coordinates of the basic IFR:

$$ds^2 = g_{00} c^2 dt^2 + 2g_{0a} c dt dx^a + g_{ab} dx^a dx^b = g_{ik} dx^i dx^k. \quad (18)$$

Here the coefficients of the 4-metric are defined in terms of γ_{ik} as:

$$g_{ik} = \gamma_{i'k'} e_i^{i'} e_k^{k'}. \quad (19)$$

The description of the same events in an extended NFR, thereby, turns out to be dependent on the choice of the basic IFR, which makes the introduction of such NFRs an ambiguous procedure. However, this ambiguity is the same as in SR, where, at a spacelike interval, the simultaneity of two events is relative. The hypersurfaces of simultaneity of different basic IFRs are related by transformations from the Poincaré group, and therefore the coordinates of events in two extended NFRs on their basis will also be related similarly. Namely, at the transition from one extended NFR to another (with a different acceleration of their LFRs) the coordinates (cdt, dx^a) are transformed together with the transformation of the metric g_{ik} , but these transformations will consist the local Lorentz transformations between the LFRs and translations. Parametrizations, do not changing the configuration of the LFRs, are also possible, but they do not change the physical picture and play only an auxiliary role in describing physical phenomena.

Here, the metric g_{ik} of the NFR depends, in addition to the configuration of curvilinear coordinates of basic IFR, also on what forces accelerate the LFR of the given NFR and velocities due to acceleration beginning from the moment t_0 . The appearance of relativistic contraction of scales and dilation of proper times of the clocks of accelerated LFRs w.r.t. the basic IFR determine the nontrivial connection and curvature, which are expressed through the metric g_{ik} and its derivatives [5]. The presence of two metrics - the metric γ_{ik} of the basic IFR and the metric g_{ik} of an extended NFR allows further use of the bimetric formalism [6] and obtain the corresponding connections and curvature tensor at describing the spacetime by using GFRs in the form of an extended NFR. In this case, the curvature of spacetime is equal to zero, while the curvature of the GFR is nonzero in the presence of a field of accelerations of LFRs of the NFR.

Thus, to construct the GFR in flat spacetime, corresponding to an extended NFR, the positions of the LFRs included in its composition must be fixed simultaneously at the moment of time t of the basic IFR K , considering the scales and clocks K as basic ones. The

construction of such GFR from LFRs on the hypersurface of simultaneity of the basic IFR, thus, means a consistent from the physical point of view method of globalization of the LR. At describing a system of particles, it is natural from a physical point of view to choose as the basic IFR, where the center of inertia of the system of particles is resting, and at describing phenomena in a certain field - IFR, where the center of inertia of the field source is resting. This allows us to correctly go beyond the IFR, describing physical phenomena also in an extended NFRs, which allows us to separate the physical part of GR. For a spherically symmetric physical system, the distinguished global IFR is the one where its center of symmetry is at rest, since only in it the system remains spherically symmetric, and in all others it will look ellipsoidal due to the contraction of lengths along the relative velocity.

Note that the GR formalism, based on the principle of general covariance, allowed the use of arbitrarily moving LFRs, both in flat and in curved spacetime. But many of them were non-physical, i.e. physically unrealizable, since they could not be related with physically realizable LFRs by the local Lorentz transformations. In a flat spacetime, it is possible to unambiguously restrict the GR formalism to sets of only those LFRs that are physically realizable, i.e. correspond to the LR, and therefore can be part of the GFR, joining simultaneously coexisting LFRs.

Further, the same consistent physical treatment will be used to describe events in the gravitational field.

C. Physics in the GFR resting in a gravitational field.

GIR in flat spacetime in the case of non-inertial GFRs is based on two facts related to the physical properties of systems with accelerations. Firstly, the GFR is formed from a set of LFRs simultaneously coexisting at a given time of the basic IFR, i.e. is built on its hypersurface of simultaneity. The second fact is the validity in these GFR of the locality principle, expressed by the local Lorentz invariance, which restricts the principle of general covariance of GR, leaving only LFRs related by the local Lorentz transformations.

In a curved spacetime, GIR as a physical part of GIR, should lead to the theory of gravity. As in GR, in GIR we proceed from the principle generalizing experimental facts, the equivalence principle, in accordance with which the phenomena in the LFR resting in the gravitational field are equivalent to the phenomena in the LFR moving with acceleration in flat spacetime [2]. It follows from this principle that in a gravitational field, as in accelerated LFRs, LR is locally valid, and therefore neighboring LFRs, resting in the gravitational field, are related by the local Lorentz transformations. The spacetime interval in the gravitational field therefore has the same form as (16):

$$ds^2 = c^2 dt^2 - dx^{a'} dx_{a'}. \quad (20)$$

Although there are no extended IFRs in the gravitational field, nevertheless, the GFR can be built on the hypersurface of simultaneity of the distant IFR, where the field source is resting.

Thus, in the GIR, to take into account the effect of gravity itself, there is no need for moving LFRs and here it is enough a set of LFRs resting relative to the field source, where the effects of gravity are manifested in their pure form without kinematic distortions. Thus, we can construct of GFR based on the rest frame of the center of inertia of the field source.

The simplest are cases of static fields, where a light signal travels between two points in forward and backward directions in the same time interval. This makes it possible to synchronize the coordinate (or world) clocks according to the Einstein procedure, after which they will everywhere show the same time, the time of the distant IFR or the *world time* t . The set of synchronized and mutually resting world clocks defines the hypersurface of simultaneity. Further, in any field with a certain symmetry there is a surface or at least a line outside the source, along which the standard scales either remained the same as in the distance, i.e. were not influenced by gravity, or this influence easy to take into account.

In a spherically symmetric field, this takes place along 2-spheres around the center, and this allows us to introduce the Schwarzschild coordinates t, r, φ, θ . In the case of a rotating source, the standard scales along the rotation axis do not have kinematical distortions, while the influence of gravity is similar to the spherical case, which allows us to introduce an analogue of the Schwarzschild coordinates with replacing of spherical by spheroidal coordinates (Boyer-Lindquist coordinates). If the lengths of all standard scales are expressed through the unchanged scale, as in (17), then the situation will be similar to the non-inertial GFR in the flat space of time, but now the contraction of standard scales and the dilation of proper times occur not due to motion in flat spacetime, but due to resting in the curved spacetime, i.e. in the gravitational field.

In this case, the spacetime interval takes the same form as in (18), and in the static case, the non-diagonal component of the metric can be excluded:

$$ds^2 = g_{00}c^2 dt^2 + g_{ab}dx^a dx^b = g_{ik}dx^i dx^k. \quad (21)$$

In the general case of non-static gravitational fields, the definition of the hypersurface of simultaneity is not so simple, although it is possible. For this, the worldlines of particles of an extended object, parametrized by their proper time τ and world time t , are obtained from the solutions of the field equations for the metric and the equations of motion for particles, and then along the worldlines we determine the events marked with the same values t , and determine the structure of matter on the hypersurface of simultaneity $t = \text{const}$. In the next Section, this will be shown in the case of some most important physical systems where gravity dominates.

Thus, the GIR in the rest frame of the source of the gravitational field can be formulated by means of the GR formalism, where the time coordinate x^0 is not an arbitrary parameter, but is the GFR world time t , i.e. $x^0 = ct$. The equations for the gravitational field, i.e. Einstein's equations are derived in the same way as in GR, from the corresponding action function. Determination of the field energy in the GFR with $t = \text{const}$. is physically correct and therefore in GIR there is no problem with the energy of the gravitational field.

Unlike GR, in the GIR the spacetime coordinates remain physical, and the transformations between different GFRs are performed as a chain of the local Lorentz transformations between the corresponding LFRs. At the same time, there remains the possibility of formal reparametrization of physical coordinates and metrics in a gravitational field, similar to canonical transformations, which was previously taken seriously in GR as a transition to arbitrary FRs.

Thus, the GIR, as GR in terms of physical coordinates given on the hypersurface of simultaneity of the basic LRR, preserves the physical part of GR, including all experimentally observed consequences, and excludes the non-physical part of GR.

D. *Physics in GFR moving in a gravitational field.*

The transition to moving LFRs in the gravitational field can be performed by the Poincaré group transformations for a distant IFR, and by the local Lorentz transformations and translations in the vicinity and inside the source for the transition to a new hypersurface of simultaneity with a new world time $t \rightarrow t'$.

In this case, the former static metric becomes non-static, which complicates the situation with the synchronization of the world clocks. If, in the case of a uniform field, there are still some possibilities of maintaining the simplicity of synchronization, then for other configurations, such as spherical or axial symmetries, such transformations sufficiently complicate the description so that it makes no sense to do them for a single source.

The situation is completely different for a binary system, especially when two sources rotate around a common center of inertia. Here the GFR with its world time should be built in the rest frame of the center of inertia of the binary system and the stationary field created by them is then described by an analogue of the Kerr metric.

Thus, in practice the introduction of moving LFRs around the center of inertia of the source refers to the LR and has only a purely local meaning, while the gravitational field as a whole and in its pure form is described only in GFRs resting relative to this center. In other cases, it is necessary to separate kinematic distortions from the full metric and only then draw conclusions about the structure of the field and matter, which, significantly complicating the description and calculations, will ultimately give the same physical picture that would follow from the description in the GFR resting w.r.t. the field source.

3. Description of phenomena in the gravitational field in global relativity

At describing physical phenomena in a gravitational field in GR, the Hamiltonian formulation, the formulation of the Cauchy problem, as well as different types of bimetric formalism were used, where the spacetime was separated into space and time [5,6]. At such separations of hypersurfaces, the formalism of GR is transferred to the GLR if it is a hypersurface of simultaneity, and the time coordinate is world time on this hypersurface.

Examples of such a description in weak fields are well known, in particular at describing the observable effects and gravitational waves. For this reason, below we will give only some new examples of the treatment of phenomena in strong fields, when the description on the hypersurfaces of simultaneity significantly changes the physical picture, and also leads to the elimination of a number of misunderstandings and errors in previous treatment.

A. Gravitational regularization of quantum fields.

In the standard formulation of quantum field theory, there are ultraviolet divergences in higher-order corrections of the perturbation theory described by loop diagrams. Regularization of loop integrals at finite distances (and energies), when the corrections to masses and charges are still small and the series of perturbation theory converge, made it possible to describe with high accuracy all known experiments. The problem, thus, was in finding a physical mechanism making the regularizations finite so that the corrections to masses and charges remain small.

In the paper [7] it was shown that such a mechanism exists and this is the gravitational time dilation, one of the basic effects of GR confirmed in the experiment. The standard formulation of the quantum field theory included high energy quanta, but did not take into account their gravity, and in attempts to take into account gravity it was considered only as one of the fields. But the external gravitational field of quanta also slows down the local proper times relative to the world time t of distant observers.

As a result, at the Planck length, which is the gravitational radius of Planck energy quanta, all processes freeze in terms of the world time of distant observers and therefore do not contribute to the probabilities of particle physics processes. The freezing of quantum fluctuations means an extremely strong redshift of frequencies up to practically vanishing, i.e. taking into account the effects of gravitation of quanta leads to the gravitational regularization of loop diagrams at the Planck length.

The nonlinearity of the fields increases the gravitational effects, which means that freezing will begin at even larger distances, which further suppresses the contributions of high energies. The finiteness of higher-order corrections of perturbation theory at small distances also makes models with nonrenormalizable fields consistent if the corrections are small over the Planck length. This is the case for gauge fields and quantum gravity, where the invariant cutoff of integrals at the Planck length (or energy) gives upper limits for loop corrections, which turned out to be small and therefore, in these theories, the perturbation theory converges.

Thus, quantum field theory became final and consistent when it was adapted to the already known and well-proven principles of physics, the requirements of which had to be taken into account from the very beginning. In this regard, there is no longer a need for searching and testing very radical hypotheses, and this greatly simplifies and significantly improves the situation in particle physics.

B. Collapse of a dust shell and a uniform dust star.

The physical picture of the collapse of stars in the framework of the GIR was presented in papers [8,9]. The main fact here is that the parts of a star as an extended object must coexist simultaneously, i.e. defined on the hypersurface of simultaneity $t = \text{const.}$, matched on the surface with the hypersurface of simultaneity of the outer region.

The Newtonian picture of the stellar collapse was simple: if the mass of a collapsing star several times exceeds solar mass, then its surface crosses the gravitational radius of the star at the speed of light and falls with an even greater speed to the center, where, together with the rest of the star's matter, forms a singularity, an infinitely dense state.

In GIR, the gravitational dilation of local proper times τ w.r.t. t leads to another picture of the evolution of the star's surface. In terms t , the surface freezes outside the gravitational radius and never reaches it, while near the gravitational radius τ practically freezes. The worldlines of particles on the surface describe in terms t every moment of their existence in the real world and therefore give a complete picture of the evolution of these particles, as well as of the surface as a whole. This treatment led to the theory of frozars (from "frozen star"), which describes the picture of the complete freezing of the structure of stars in terms t .

The formation of a frozar clearly demonstrates the collapse of a thin dust shell. The shell, in terms of t , freezes outside its gravitational radius, its interior remains flat, and the test particles inside it freeze where they were before the shell freeze. The shell transforms into a *hollow frozar*, an object completely frozen due to its strong gravity. The hollow frozar does not have an "event horizon", since the shell freezes outside the gravitational radius, and there is no singularity at the center, since the inner region remains spatially flat and there are no shell particles there. The equations of motion give the worldlines of the shell particles for any moment $t < \infty$, at each moment the shell remains outside the gravitational radius, and the area occupied by the shell becomes *a place where nothing happens*.

A star is an extended object, a collection of simultaneously coexisting particles and fields, and therefore its structure as a whole is determined without kinematical distortions only in the rest frame of its center of inertia. Outside a spherical star, its gravitational field is static, and for the inner layers of the star, solutions of the field equations and equations of motion allow one to relate the proper times of particles in layers to the proper times on the surface. By selecting along the worldlines of particles of layers the values of τ corresponding to events simultaneous with the surface, one can describe the instantaneous structure of the star at the appropriate moment t .

In the case of a spherical dust star, the density of which is locally uniform, and the particles fall radially with parabolic velocity, the exact solution of the Einstein equations was found in 1939 by Oppenheimer and Snyder (OS). The OS solution consisted in transforming the local Friedman solution in terms τ (in Tolman's form) into a global solution in terms t and described the global structure of a dust star as a set of simultaneous events. Later, exact solutions for elliptical velocity were found by two other methods by O. Klein (1961) and S. Weinberg (1972). The complete solution of the problem of the collapse of a homogeneous dust star using these three methods (OS, Klein, and Weinberg) for three selected velocities (parabolic, elliptical and hyperbolic, i.e. $k = 0, \pm 1$) was given in [8,9] and took the form of frozar theory, which predicted a number of observable effects.

These exact solutions showed that not only the surface of the star freezes in terms t , but its entire volume as well. The center of the star freezes before other layers, after which the freezing quickly reaches the surface and the entire structure of the star freezes. This means that the gravitational time dilation is the physical mechanism for stopping the collapse in terms of t . Each of the inner layers freezes at its effective gravitational radius created by the matter inside this layer (taking into account the influence of the outer layers). Before freezing, the density of sufficiently massive stars is lower than that of a neutron star, and therefore, if they

did not explode before, then the collapse of such stars occurs similarly to a dust star with the formation of a frozar.

Thus, the collapse of stars in the GIR at any finite moment t , the time of the external world, leads to a frozar, an object with an almost homogeneous and practically frozen internal structure. Frozar is a star with a surface outside the gravitational radius, the internal structure of which is frozen in the state in which it was before freezing. Frozars do not have a horizon or singularity and lead to a number of observable effects, and the area of space they occupy is "a place where nothing happens."

Unlike neutron stars, when matter falls on a frozar, there will be no flash of radiation, since the matter and the process of its fall quickly freeze near the surface. Therefore, for an outside observer, the fall of matter on the frozar will be very "quiet". This and a number of other properties of frozars are consistent with the observed properties of well-known compact objects that are "candidates for frozars."

The proper time of a particle on the surface of a rotating star is slowed down in relation to t both gravitationally and kinematically, while the non-radial part of the velocity of these particles is maximal at the equator and equal to zero at the poles. The local speed of surface particles only asymptotically approaches the speed of light at any finite moment t . The surface freezes, being an oblate ellipsoid, processes in the inner layers also freeze, and the star turns into a *frozar with angular momentum*. As a result, in the static frame of reference, the frozar surface with angular momentum ceases to be compressed radially, remaining outside the outer gravitational radius of the Kerr metric, which was previously considered as the outer boundary of the ergosphere. The meaning of this boundary turns out to be that here the local velocity of surface particles would reach the speed of light. Since the metric inside such a frozar is given by a material solution and is not the Kerr metric, such a frozar does not have an ergosphere. As a result, there will be no effects associated with the possible existence of the ergosphere, such as the extraction of rotational energy when particles enter the ergosphere by subsequent release.

The matter accreting on the frozar, falling almost radially, quickly freezes on the surface, and the transverse size of the fall area does not grow and the matter flattens radially without spreading over the surface. This leads to non-uniformity on the frozar surface, similar to massive anomalies (mascons) on the lunar surface. When matter falls from the disk, the inhomogeneity is maximal near the equator. In the case of a radial falling of a neutron star onto a more massive frozar, the neutron star will flatten and freeze on the frozar surface in the form of a thin local "haircut", a large mascon. It is also possible for a neutron star to explode due to lateral compression at falling on a frozar, which can lead to the observed powerful burst. The possible heterogeneous landscape distinguishes frozars from white dwarfs and neutron stars.

Inhomogeneity of the distribution of matter on the surface, i.e. heterogeneous landscape, leads to inhomogeneities of the frozar gravitational field over these areas, which lead to the observed effects. They can be detected using gravimetric methods, as well as irregularities in the redshifts of matter around a frozar, as well as by deviations in the shadow of a frozar. Perturbations of redshifts and orbits of objects around the frozar are also observed, including additional orbital precessions and disk inhomogeneities. Variations in the disk brightness due to the inhomogeneous structure of the frozar can possibly be separated from other contributions, especially for supermassive frozars, in which such structural anomalies will be strong.

In Newtonian theory, when two compact spherical objects merge, a more massive compact object is formed, which is also almost spherical. However, in the GIR, two or more frozars cannot merge at approaching and freeze at some distance from each other, forming a frozen cluster of two or more frozars. If two frozars of equal mass fall radially towards a common center of inertia, then, since the gravitational radius of the system is twice the gravitational radius of each frozar, the surface of each frozar remains outside the gravitational radius of the system due to the freezing of all processes, including falling, in terms of t . Thus, in the GIR, two frozars cannot merge and only their sticking together occurs. The same situation, even more illustrative, takes place for three and four symmetrically located frozars,

when the gravitational radius of the system is approximately three to four times greater than the gravitational radius of each of the frozars. At the same time, a frozen cluster of frozars forms a gravitational crystal, where the distances between frozars are several times larger than the size of each of them. Sticking together of frozars forms the largest inhomogeneity in a cluster of frozars and leads to the observed effects that are stronger than the effects associated with mascons on the surface. In this case, the sticking together of two frozars leads to a shorter and weaker flash of all types of radiation, including gravitational waves, than in the case of merging.

At quantum fluctuations of fields in a static gravitational field, the worldlines of particles are ordinary both inside and on the surface of the star. In this case, the energy of the virtual particle-antiparticle pair is positive. Therefore, in GIR, there is no quantum production of real pairs of particles from the vacuum in the field of a static frozar. But at vacuum fluctuations of quantum fields in the vicinity of a frozar in the presence of a thermostat or an external field, one of the quanta of the pair can fall on the frozar, and the other can leave its vicinity. In this case, the freezing of some of the quantum fluctuations in the frozar leads to the appearance of effective entropy and temperature. But, in contrast to the hypothesis of "quantum evaporation" of collapsed objects, at such a particle production due to the energy of the external field, the frozar mass increases due to the absorption of particles of positive energy. Therefore, in these cases we should not talk about "evaporation", but about "condensation" on the frozar of one of the particles of any pair.

C. Relativistic cosmology at local and global scales.

There were a number of problems in relativistic cosmology that not only remained unsolved, but were not recognized as problems either. The GIR allows one to identify them, which further allows one to find their solutions. In particular, the Friedmann models are based on the assumption that the rate of proper times is constant during the expansion. Therefore, due to the need to take into account the contributions of the stretching of wavelengths and the Doppler effect, *the double redshift paradox* arises [10].

In static space, photons from sources receding from us will come to us with Doppler redshift. Moreover, in our rest frame, this displacement is already present at the beginning and does not change further, i.e. photons will arrive at with initial Doppler redshift regardless of the distance. If space expands, but the source is at rest relative to us (does not comoving the expansion), then photons from it are emitted without Doppler redshift. Along the way, the wavelengths of these photons will increase due to the expansion of space and the photons will arrive at with stretching redshift. This purely cosmological redshift increases with distance. These are two independent mechanisms of the cosmological redshift: the first, the Doppler effect for receding ones, exists already at the beginning, and the second, stretching of wavelengths for any sources, arises during propagation.

In the expanding space, the speeds of objects comoving the expansion increase with distance. If we introduce a source near such an object, resting relative us (not comoving the expansion), then the photons from it will come without the Doppler effect and only with stretching of the wavelengths. And photons from a receding source are emitted with the Doppler redshift at first, and then along the way acquire an additional redshift due to stretching of their wavelengths. At low speeds, the contribution of both types of redshift is the same, which was the reason for the confusion in the treatment of the nature of the redshift. If the velocities are not small, then the Doppler effect is relativistic and differences arise.

Thus, the cosmological redshift of the objects comoving the expansion contains the contribution of both mechanisms, and not one of these effects, as it was assumed in the Friedmann models. Then, under the assumption that the wavelength first coincided with the wavelength of the photons here with us, the observed redshifts correspond to the contribution of only one of these mechanisms. Thus, the theory predicts two equivalent types of redshifts, and the interpretation of observations with this assumption shows the presence of only one of them. This is *the double redshift paradox*.

In the papers [10], it was shown that agreement between theory and observations is achieved not by ignoring one of these two independent physical phenomena, which is erroneous and led to the contradictions in the theory, but by taking into account the third mechanism admissible in GIR, which compensates for the contribution of one of the two types of redshift

It follows from the cosmological principle that the wavelength of a photon when emitted at a distance in an earlier epoch coincides with the wavelength of the same photon in our place only at the same early epoch (without taking into account the Doppler effect). The wavelengths in different places in different epochs are not limited by the cosmological principle. In the GIR there is freedom in choosing the rate of proper times in different cosmological epochs and the choice should be based only on observational data. If we assume that in addition to two mechanisms of cosmological redshift, the Doppler effect and stretching, there is also a third mechanism - the violetshift for photons emitted in earlier epochs due to a faster rate of proper times, then this new type of shift is compensated for in the way by the stretching redshift and only the Doppler redshift is observed.

Thus, to solve the double redshift paradox, the rate of proper time in previous epochs had to be faster than now. Choosing the rate of our proper time as a standard (since the signals from previous epochs are compared with the signals in our time), we come to the model of *slowing time cosmology*, where there is no double redshift paradox.

Aberration, the change in direction to a light source and its apparent luminosity due to the relative speed of the observer, is one of the main optical effects in astronomy, which eliminates optical distortions. In relativistic kinematics, the Doppler effect and the aberration of signals from receding sources are determined by the same factors, i.e. the presence of the Doppler effect also leads to aberration. These two effects affect the properties of the flux of photons together - the first reduces the frequency of photons, and the second - the number of photons within a given solid angle. At an increase in the solid angle of a cone with the same number of photons, the apparent luminosity decreases. This decrease in the density of the light flux coming from receding objects must be taken into account both in the expanding universe and due to aberration (arising, like the Doppler effect, at the moment of emission), receding sources will become dimmer than sources resting at the same distance.

Observations already in the linear part of the dependence of redshifts on distance reject models with the Friedmann metric, leading to a double redshift and agree only with the slowing time cosmology with a single final redshift. As shown in [10], the new relation "distance modulus-redshift" is consistent with observational data and describes them without dark energy or other speculative hypotheses.

In the slowing time cosmology, the cosmological constant, and hence the dark energy, are absent, and the Einstein equations with the energy density of matter are sufficient to describe the observations. This achieves agreement between cosmology and particle physics in the problem of vacuum energy. Slowing time cosmology also leads to a number of new consequences for the early universe and allows solving the known cosmological problems.

The infinity of space in Newtonian cosmology led to a number of problems that were practically insoluble. Relativistic cosmology also has models of an "open" universe of infinite volume (with flat and hyperbolic spaces), in which the same problems arise. At first, the possibility of a "closed" universe of finite volume in the form of a three-dimensional sphere seemed to be one of the main advantages of relativistic cosmology. Moreover, as it turned out later, it is more natural to explain the Big Bang in a space of a finite volume than in an infinite space. However, in Friedmann's models it was not possible to match the parameters of the closed model with observational data, and such models were practically out of sight of researchers. But in slowing time cosmology, the curvature of space in our epoch does not play such a decisive role as in Friedmann's models. In this case, the model of a closed universe turned out to be quite admissible for the parameters that describe the observational data. Therefore, the possibility of restoration of models of the finite volume universe in GIR can become one of the most radical changes in the physical picture of the world.

Conclusion

Thus, both local and global relativistic effects are universal and therefore are described by separate theories - LR and GIR. In terms of physical coordinates, the local parts of SR and GR, restricted by LFR related by local Lorentz transformations, form LR, and their global parts, also in extended GFR, built from simultaneously coexisting LFR, form GIR. Thus, LR and GIR turned out to be universal theories, each of which describes physical phenomena in a unified way both in flat and curved spacetimes.

This made it possible for a purely physical description of the phenomena of quantum and gravitational physics, using the methods of these two theories, which removes a number of problems in these areas of physics, which for a long time did not have an unambiguous solution in the framework of GR.

The history of the evolution of relativistic concepts and various applications of LR and GIR will be described in more detail in forthcoming publications and in the book [4].

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